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THE MAGNETIC SEPARATION OF ABSORPTION LINES IN CONNECTION WITH SUN-SPOT SPECTRA[1]

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1. As a consequence of the intimate connection between emission and absorption, there exists, closely corresponding to the magnetic separation of emission lines, a magnetic division of absorption lines. The dark lines which appear in a continuous spectrum, if a beam of white light traverses an absorbing flame, are divided and polarized under the influence of magnetic forces in exactly the same way as the emission lines. This correspondence between emission and absorption was shown to exist in some of the earliest experiments on the subject by one of the present authors. Our knowledge of emission spectra under magnetic influence has since been extended considerably. The experimental study of the inverse effect, i.e., the magnetic division of absorption lines, has however advanced less.

After the first experiments of the first-named of the authors of this paper, the change of absorption lines in a magnetic field was studied by König[2] and Cotton;[3] Righi[4] made an elaborate study

[1] Paper presented in three parts to the Royal Academy of Sciences of Amsterdam. From the Proceedings of the Meetings of Jan. 29, Feb. 26, May 28, and June 25, 1910.

[2] *Annalen der Physik*, **62**, 240, 1897.

[3] *Eclairage électrique*, 5 et 26 mars 1898.

[4] "Sul fenomeno di Zeeman nel caso generale d'un raggio luminosa comunque inclinato sulla direzione della forza magnetica," *Mem. di. Bologna*, 17 Dicembre 1899.

of the subject, to which we have to return later on. It contains the only investigation of the magnetic effect in a direction inclined to the lines of force. Closely connected with our subject are finally some observations by Lodge and Davies[1] on the influence of a magnetic field on flames emitting "reversed" lines.

The consideration of the inverse effect forms the basis of Voigt's magneto-optical theories;[2] and it is considered also by Lorentz[3] in his investigation of the magnetic separation in a direction inclined to the line of force.

Theory indicates different points, which may be tested by experiment. The inverse effect has become of supreme interest in solar physics, since Hale's[4] discovery that the dark lines of the sun-spot spectrum exhibit the characteristic phenomena of magnetic separation.

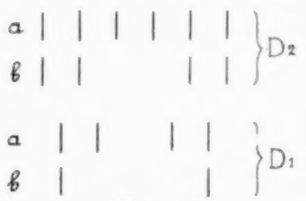


Fig. 1

The experiments we intend to describe in the present communication relate to the division of the sodium lines D_1 and D_2. Some of our results may already be found in the work of the authors cited, but in order to present the subject in a connected form it seemed necessary not to exclude these.

The facts now ascertained in combination with former results appear to be of some value in explaining peculiarities observed in sun-spot spectra. Some instances will be given later on.

2. Type and relative amount of the magnetic division of the sodium emission lines, D_1 and D_2, are given in Fig. 1.

a represents the observations when the line of sight is at right angles to the magnetic field, *b* when it is parallel to the field.

In a weak magnetic field, D_2 exhibits the triplet type at right angles to the field; the doublet type, if the light is examined parallel to the lines of force. D_1 seems to exhibit a doublet in both principal directions.

[1] *Proc. Roy. Soc.*, **61**, 413, 1897.

[2] *Magneto- und Elektrooptik*, chap. iv and the papers there cited.

[3] *Proceedings of the Royal Academy of Sciences of Amsterdam*, **12**, 321, 1909.

[4] "On the Probable Existence of a Magnetic Field in Sun-Spots," *Contributions from the Mount Wilson Solar Observatory*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

The Fraunhofer lines in the spectra of sun-spots investigated by Hale are either broadened, or changed to doublets (often incompletely resolved quartets), or to triplets. The resolutions exhibited by sodium vapor are therefore the very types of special importance to astrophysics; this and also the facility of producing sodium vapor in the magnetic field induced us to commence our experiments with this substance.

3. The explanation of the inverse effect is easily understood by means of the well-known law of resonance. If there are in a flame under the influence of a magnetic field three periods of free vibrations, then we may expect that from incident white light, vibrations of just these three periods will be taken away. The absorption is a selective one, with the peculiarity that the selection refers not only to the period but also to the direction of vibration. Consider for example the central component of a triplet which in the emission spectrum is due to vibrations parallel to the field. From incident white light, only vibrations corresponding as to period, as well as to direction of vibration with the middle component, are absorbed. Vibrations perpendicular to the field, though of the period of the unmodified line, pass unimpeded. On the contrary, white light of periods coinciding with those of the outer components is deprived of its vertical constituents only.

It will be clear from these very simple considerations what we may expect to observe with white light under the conditions of the experiment. The arrangement was the following: White light of the incandescent positive pole of an arc lamp traverses a sodium flame, placed between the poles of a Du Bois electromagnet. This light is analyzed by means of a stigmatic spectroscope having a large Rowland grating. The observations are made in the first order.

If the observation is made at right angles to the lines of force, we see in the continuous spectrum four dark components in the case of D_1, six dark components in the case of D_2, as represented for both lines under *a* in the diagrammatical Fig. 1. In order to observe all these components the field must be strong and the vapor-density adapted to the field. The groups of lines indicated by *b* are seen, if the light is examined axially. All these components, if narrow, are seen only diffuse and not black. From the considerations above

given the reason will be clear at once: each of the components absorbs only *half* the incident natural light. With very diluted vapor no absorption at all or only very weak traces of absorption are seen.

4. The introduction of a nicol in the beam, before or after the field, entirely changes the phenomenon. The absorption lines can then be seen very narrow and black. Let the observation be made at right angles to the horizontal field, then, if the nicol is placed with its plane of vibration vertical, D_1 exhibits its two, D_2 its four outer components. After a rotation of the nicol through $90°$ both D_1 and D_2 give only the two horizontally vibrating components.

Let a beam of natural white light traverse axially the magnetized vapor placed between the perforated poles of an electromagnet. Then by means of a quarter-wave plate and a nicol we may quench either the right-handed or the left-handed circularly polarized component.

A combination of a quarter-wave plate and a nicol, converting incident light into right-handed circularly polarized light, may be called a right-handed circular analyzer. The absorption line corresponding to a right-handed circularly polarized component is seen with both increased clearness and darkness by examining it with a right-handed circular analyzer. We introduce this simple matter here because there has been occasionally some confusion on this subject.

5. The behavior of horizontal and vertical vibrations may be studied simultaneously by using a calc-spar rhomb, according to the suggestion of Cornu and König. By means of it we can obtain two oppositely polarized images of a horizontal slit of suitable width, placed near the magnetic field. Right-handed and left-handed circular vibrations can be separated on the same plan by the introduction of a Fresnel rhomb between the calc-spar and the slit of the spectroscope.

It is, however, of considerable interest to examine also the behavior of the lines in natural light. A separate examination after the removal of the polarizers might be made. The vapor-density ought to be the same in both experiments. It seems difficult to realize this, in practice.

The desired end is secured more simply and surely, and with only half the labor, by adopting the width of the horizontal slit and the thickness of the calc-spar in such a manner that the two images given by the calc-spar partially overlap. We now obtain three strips; the central one exhibits the phenomena as seen without polarizing apparatus (see Fig. 2). The upper and lower strips show the influence of polarized light on the phenomenon.

The observations given in this communication have been made by the method described. By its use all particulars of the phenomenon are simultaneously exhibited; we also succeeded in photographing the essential points. Examples of our photographs are given on the accompanying plates.

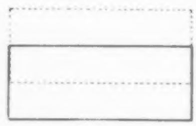


Fig. 2

6. If the absorption lines are not narrow or if the magnetic field is weak, the components of a magnetically divided line will partially overlap. This partial superposition is the cause of some peculiarities, especially manifest in the inverse effect and probably also apparent in sun-spot spectra. The nature of these peculiarities may be illustrated by a few examples. We will consider the case of the magnetic triplet and the magnetic doublet.

In Fig. 3 the curves show the distribution of intensity of the three components of a triplet, if the light is examined at right angles to the lines of force. If natural light traverses a source of light placed in a magnetic field, two black bands are seen, corresponding to the wave-length for which vertical as well as horizontal vibrations are absorbed. These black bands are surrounded by less dark parts, which absorb only one of the principal vibrations, the other proceeding unimpeded (see paragraphs 3 and 4).

If a nicol, with its plane of vibration vertical, is now introduced, two black bands are again seen. The darkest part of these components corresponds to the maximum of the curves relating to vertical vibrations. As a general rule the distance of the components exceeds that of the lines first considered.

7. Parallel to the lines of force a partial, not too small, overlapping of the components produces a black line limited by two less dark parts. This case is illustrated diagrammatically in Fig. 4.

The two components may be separated by a circular analyzer. These considerations may be applied to the magnetic division in sun-spot spectra; as a general rule we may expect that the separation of lines in spot spectra becomes more distinct and of larger amount by the use of analyzers.

The introduction of a nicol in the beam may also reveal lines invisible without analyzer. Several peculiarities observed in the distribution of intensity in spot lines remind one of the superposition phenomena now specified;[1] see 19 below.

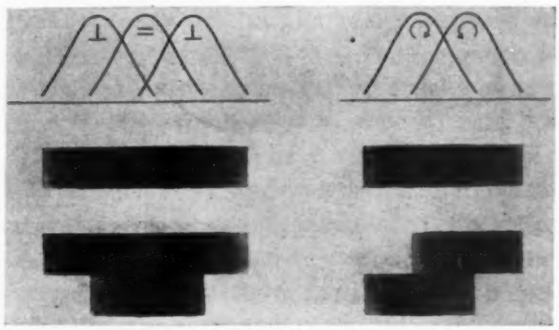


FIG. 3 FIG. 4

8. Superposition effects of nearly, though not exactly, the same nature occur if lines with the same direction of vibration are superposed and if the continuous source of light emits unpolarized light. In the more complicated divisions the superposition now specified occurs also. It is just possible that the superposition of the outer components of the sextet, type D_2, produces only dark, that of the inner and the next outer components, black lines in the continuous spectrum. It is easily seen that also in the case of the quartet, type D_1, black lines may be produced. The darkest parts may be seen somewhat nearer to the middle of the complete figure than the outer components of the quartet. It seems unnecessary to illustrate

[1] A figure equivalent to the one now given concerning the influence of superposition of magnetically divided components was already drawn for *emission* lines in Zeeman's "Doublets and Triplets in the Spectrum Produced by External Magnetic Forces," *Phil. Mag.*, **44**, 55, July 1897.

this by figures. Examples of the actions specified will be given presently.

9. Our observations and spectrograms also relate to the two principal directions (parallel and at right angles to the lines of force), and to directions inclined to the field.

In the present, first, communication, observations are discussed, relating to five different angles between the field and the direction of propagation of the beam (Voigt's ϕ, Lorentz' θ). These values are: 90°, 0°, 60°, 45°, 36°–39°. The results of the work relating to these angles have been recorded on nearly one hundred spectrograms.

10. Observations perpendicular to the field. In the upper of the three strips which are present in the field of view (see 5), the light vibrates vertically; in the lower one, horizontally, whereas the middle part relates to natural light.

Under the influence of the magnetic field we therefore see the vertically vibrating components as narrow black lines. The quartet of the D_1 line, the sextet of the D_2 line may be seen very clearly by this method. A small disturbance is produced by the narrow reversed lines due to the electric arc-light. The intensity of these lines depends upon somewhat variable circumstances of the arc itself. In some cases these lines are almost invisible, in other cases more prominent. They are to be seen on some of our reproductions; with our present subject they have nothing to do.

As regards the central strip we refer to the remark previously made, that the image of the separation must become rather indefinite, and weak (paragraph 3), because the absorption is only partial. The partial superposition of components gives, at least in the case of diluted vapor, the most conspicuous lines (6 and 7). In the case of the quartet, for example, one sometimes sees, instead of four, only two components, situated between the inner and outer ones. We made experiments with different vapor-densities. The observed phenomena may be classified under three phases:

1) The vapor is *very dilute*. The components are clearly visible in the upper and lower strips. In the central strip the absorption is either hardly perceptible (Plate XVI, Fig. 1) or the components of the quartet and the sextet are seen as separate, but weak lines (Plate XVI, Fig. 2). In this phase of the phenomenon

the great difference of definiteness of the central and outer regions is very remarkable. This contrast is still more marked with eye observation. In order to obtain good photographs, it was necessary to increase the density of the vapor above the one required for the observation of the very first trace of absorption.

2) Vapor of *intermediate* density. The components in the upper and lower strips are now no more separately visible, or only in the case of the quartet. In the central strip a superposition of the kind mentioned in 6 takes place. In place of the quartet an apparent doublet is seen, the components of which are situated between the outer and inner components of the quartet. This case is very clearly represented in Plate XVI, Fig. 3. The phenomena exhibited by the sextet (D_2 line) become rather complicated. The superposition phenomenon is often very distinct. The D_2 line on Plate XVI, Fig. 3, shows the appearance sufficiently.

3) With still *denser* vapor, the components become very broad and the magnetic change hardly visible. The *polarization of the edges* of the broad line may be recognized. This phase is represented in Plate XVI, Fig. 4. It corresponds to the emission effect as it was first discovered: a slight change of broad lines in a weak field. With still greater absorption the influence of the field becomes imperceptible. All these phases appear with great regularity. If the intensity of the field is known, it seems possible, the resolving power of the spectroscope being given, to deduce the density of the vapor from the nature of the observed phenomena.

The phenomena of magnetic division hitherto observed in sun-spots appear to fall under the second and third phases above mentioned. From measurements of spot lines, compared with laboratory experiments, Hale deduces a maximum intensity of the spot field of 4,500 gausses. Hence one would be inclined to think that the density in the layers which bring about the absorption in the sun-spot spectrum can only be small. Moreover, the non-uniformity of the field of sun-spots produces by itself a widening of the components. Light from a limited portion of the spot would give perhaps very narrow spectral lines. In view, however, of the critical remarks of Kayser[1] concerning our knowledge of the

[1] Kayser, *Handbuch der Spectroscopie*, **2**, Kap. v.

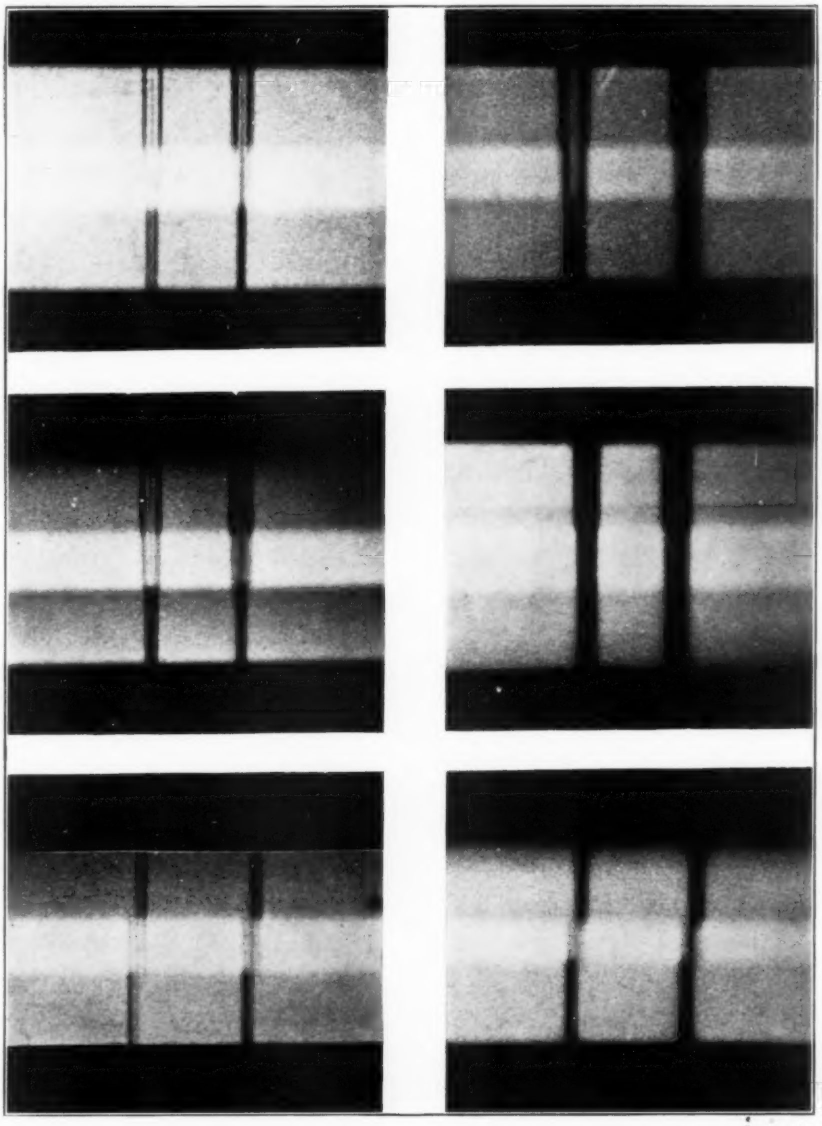
PLATE XVI

D_1 D_2 D_1 D_2

1 3

2 4

5 6



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influence of pressure and of temperature on spectra, all such considerations must be put forward with great diffidence.

11. Observations parallel to the lines of force. In the present experiment the absorbing vapor subjected to magnetic forces is placed between perforated poles.

After putting on the current, one sees in the continuous spectrum two dark bands in the case of D_1, four in the case of D_2, according to the diagrammatical Fig. 1. The absorption is incomplete also now, because for some wave-lengths only the right-handed circularly polarized light is absorbed and the reverse. In order to observe the separation and the polarization a Fresnel rhomb is placed with its principal plane at an azimuth of 45° with the horizon, a horizontal slit being placed in one of the perforated poles. The Fresnel rhomb converts circularly polarized light into plane-polarized light. By means of a calc-spar rhomb three strips are now also obtained. The first phase (very dilute vapor) is represented in Plate XVI, Fig. 5.

Vapor of intermediate density (second phase) exhibits the superposition phenomena mentioned in 7 and 8 and diagrammatically illustrated by Fig. 2. In the central strip *one* line, at the position of the unmodified one, surrounded by feebly absorbing regions, is seen. Plate XVI, Fig. 6, shows these lines for the doublet and the quartet; especially with D_2 the effect is very marked.

12. Observations in directions inclined to the field. According to Lorentz' elementary theory of magnetic division one generally observes in a direction oblique to the lines of force by an angle θ a triplet with elliptically polarized outer components.[1]

The ellipse, which characterizes the state of polarization of the components with period T_0+v, is the projection on the wave-front of the circle perpendicular to the field in which the electron with period T_0+v is moving. v is a small quantity. The direction of the motion of the moving electron also determines the motion in the ellipse. The ratio of the axes is as 1 to cos θ. For the other outer component with period T_0-v, the same reasoning holds, *mutatis mutandis*. The central line with the unmodified period T_0 always remains linearly polarized. The vibrations of the middle

[1] Cf. Righi, *op. cit.*

component are in the plane determined by the ray and the line of force and the amplitude of the vibrations is proportional to sin θ.

If we put $\theta=0$, i.e., in the case of the longitudinal effect, only circular motions remain.

All this applies to very narrow spectral lines in a strong field, the distance of the components being much greater than their width. According to Voigt and Lorentz we must expect some interesting peculiarities if this restriction be discarded. We return to this point later on. As a general rule the deductions from the elementary theory are verified. Also in the case of the quartet and the sextet the outer components become elliptically polarized, as has been observed already by Righi.[1] In contradiction with the elementary theory, though not strictly applicable to the case, is the very slight diminution of intensity of the middle components of the quartet even for $\theta=45°$.

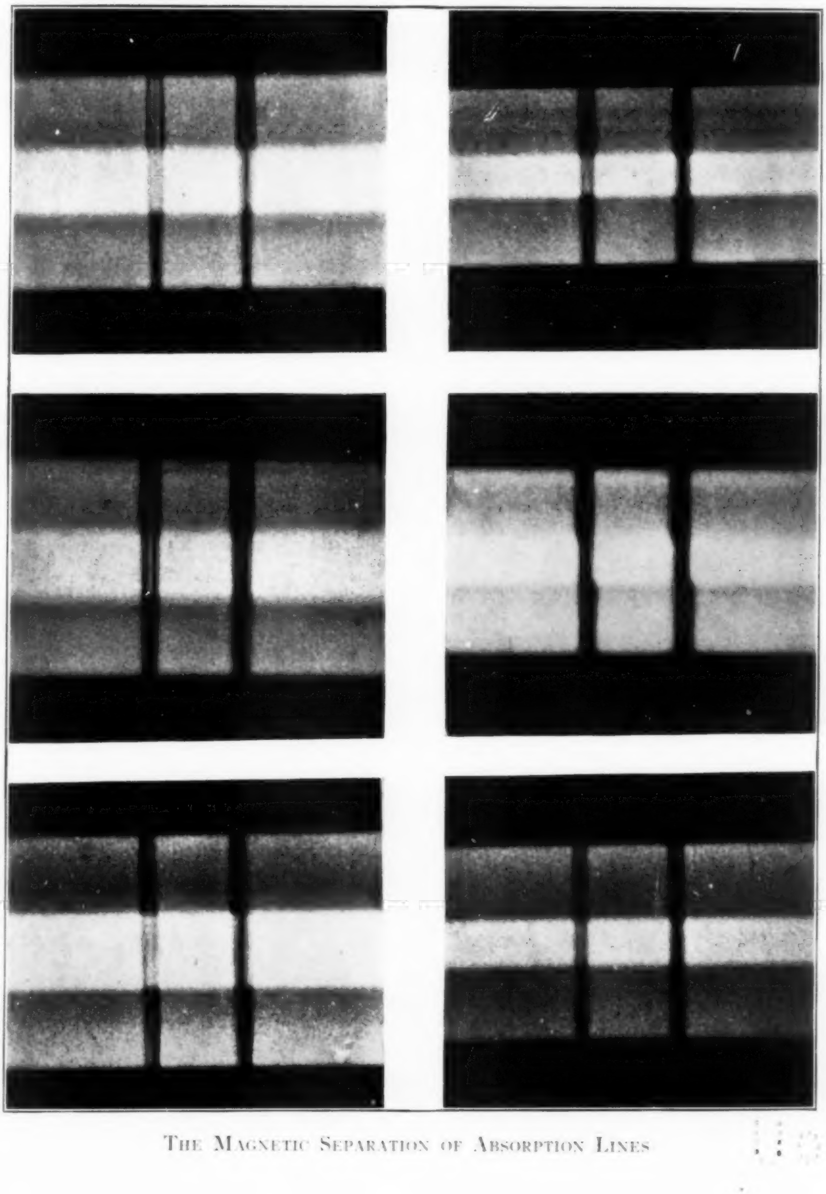
13. Observations at $\theta=60°$. If the observation is made with a calc-spar rhomb, the image remains as with the transversal effect. Yet the presence of elliptic polarization ought to manifest itself by the appearance in the lowest stripe of lines, corresponding to the outer components. With very dilute vapor and with that of intermediate density, practically no trace of it is seen. Plate XVII, Fig. 7, shows the first phase with dilute vapor, Fig. 8 the second phase with denser vapor. Only traces of absorption, indicative of elliptic polarization, can be seen near D_2, Fig. 8. The ellipticity is, however, undoubtedly proved by means of the Fresnel rhomb, placed with its principal plane at an azimuth of 45° with the horizon. Fig. 9 shows the appearance.

The outer components of the quartet toward the red or toward the violet, dependent upon the stripe and the direction of the field, are now considerably weakened; in the case of the sextet they have vanished altogether. All this proves the elliptical polarization of the outer components. For, if the polarization were linear, as might be inferred from observations with the calc-spar alone, then the observation with calc-spar and rhomb combined ought to show

[1] Righi's observations (*op. cit.*) all refer to an angle of nearly 55°, the angle at which according to the elementary theory the three components of the triplet are of equal intensity.

PLATE XVII

D_1 D_2 D_1 D_2



7 60°

10 45°

8 60°

11 45°

9 60°

12 30°

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no difference between the highest and lowest stripe. The light of all plane-polarized components would issue circularly polarized from the rhomb, and, the calc-spar making no selection between right-handed and left-handed polarizations, the components toward red and toward violet would all be alike. Such a condition is disproved by photographs such as Fig. 9, Plate XVII.

14. One point must be considered somewhat more in detail. What is the reason that the ellipticity is not shown by the calc-spar rhomb alone, whereas its existence is most clearly demonstrated by means of the Fresnel rhomb?

Let an elliptic vibration with vertical axis b, horizontal axis a, be incident upon the rhomb, the principal plane of which is at an azimuth of 45°. It is easily proved that the elliptic vibration issuing from the Fresnel rhomb has its axes in the same direction as the original motion and a ratio of the axes $\frac{a_1}{b_1}=\frac{b-a}{b+a}$, the original ratio being $\frac{a}{b}$. If a be small in relation to b (an elongated ellipse), then the light issues from the Fresnel as a more circular vibration, which is more easily analyzed. It depends upon the magnitude of a whether $\frac{a}{b}$ is *greater* or *less* than $\frac{b-a}{b+a}$.

We distinguish the following cases:

(1) a very small, then $\frac{b-a}{b+a}>\frac{a}{b}$.

(2) $a=0.414\ b$, then $\frac{b-a}{b+a}=\frac{a}{b}$.

(3) $a>0.414\ b$, then $\frac{b-a}{b+a}<\frac{a}{b}$.

We shall apply these results to the interpretation of our observations. Two cases dependent upon the magnitude of a are of principal importance.

In the first case we can observe the effect of both the axes of the ellipse by means of the combination of the Fresnel rhomb and the calc-spar (*this is the case of the quartet*) (D_1, Fig. 9), whereas without Fresnel rhomb no effect of the small axis is visible. In the second

case the effect of the small axis becomes apparent by the use of the calc-spar, whereas its existence cannot be demonstrated with the Fresnel, the value of $\frac{b-a}{b+a}$ being too small. *This case is represented by the sextet* (D_2, Fig. 9, Plate XVII).

If the observation is made by means of the calc-spar rhomb, we indeed see with dense vapor new components in the lowest strip (see Fig. 8, D_2). The theoretical import of this result will be discussed on another occasion. After introduction of the Fresnel rhomb the component to the left of the central line (small axis of the ellipse) remains invisible (Fig. 9, D_2, lower strip). Hence we may conclude that at the angle now investigated, the ellipticity of the outer components of the *sextet* (the ratio $\frac{a}{b}$) exceeds that of the quartet (and is also larger than 0.414).

15. Observations at $\theta=45°$. The photographs taken with the calc-spar alone show very clearly the ellipticity of the outer components. With vapor of intermediate density the phenomenon is already very marked, especially in the case of D_2 (Plate XVII, Fig. 10). Very remarkable is the slight diminution of intensity of the inner components of the quartet. According to the elementary theory the intensity of the central component of *a triplet* ought to have diminished already to less than *half* the original value.

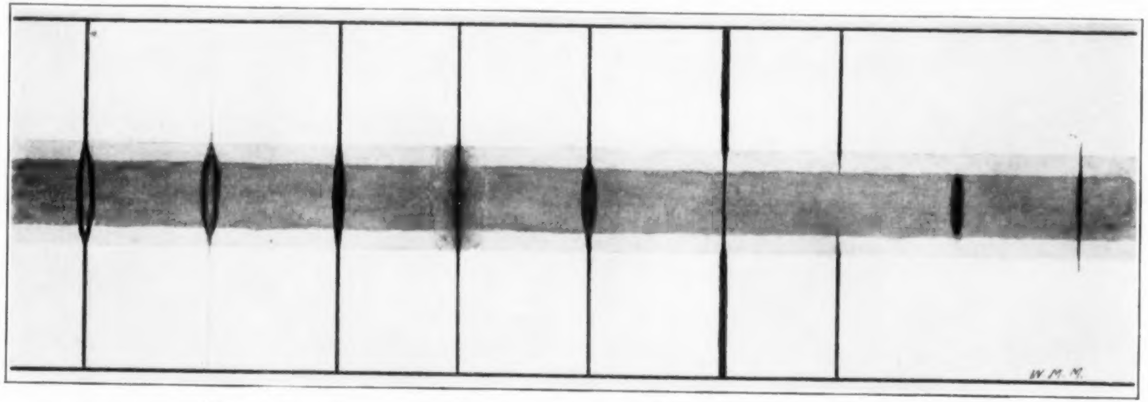
16. If a Fresnel rhomb combined with a calc-spar rhomb is introduced in the beam, one of the components of the quartet also entirely disappears. At an angle of 60° this was the case only with the sextet (Plate XVII, Fig. 11).

17. Observations at $\theta=39°$. The elliptic polarization tested by means of the calc-spar rhomb is very marked, even with dilute vapor (Plate XVII, Fig. 12, Plate XVIII, Fig. 13). The inner components of the quartet are now decidedly less intense than the outer ones. Plate XVIII, Fig. 13, especially shows the smaller intensity of the components of D_1 in the lower strip. Indeed, they are unmistakably thinner than those in the upper strip.

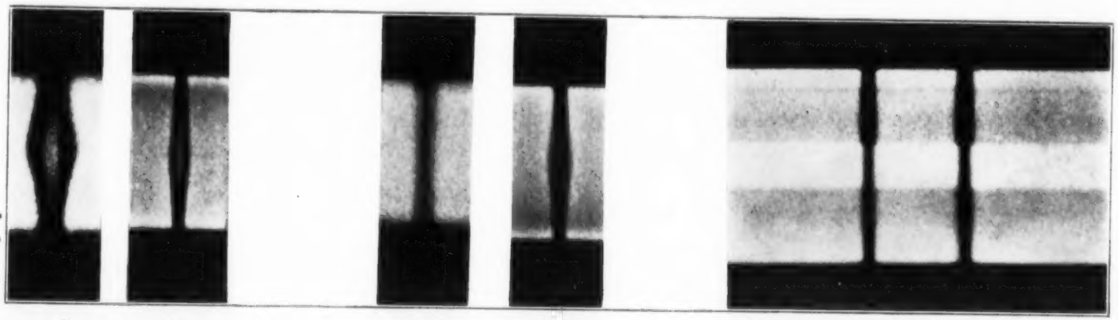
18. According as the angle between the ray and the lines of force is diminished, the intensity of the field must diminish at the

PLATE XVIII

14. Types of Sun-Spot Lines (W. M. Mitchell)



a, *b*, widened lines with centers reversed bright; *c*, widened and weakened line; *d*, winged line



a′, *b′*, *e′*, types of magnetic resolutions in non-uniform fields; *d′*, superposition of magnetic components

same time. In order to make it possible for the rays to traverse the field under smaller angles the vertex semi-angle of the cones must deviate more and more from the theoretical optimum of nearly 55°. The decrease of the magnetic separation is clearly shown in our photographs. We intend to communicate on another occasion experiments under smaller angles θ and to enter upon some details concerning the case in which the components of the triplet are not neatly separated. Some measurements of the ellipticity of the components will also be given. On the present occasion we intended to give only a general survey of the inverse effect, illustrating it by some particular cases.

19. Types of separation in spot and laboratory. In one direction we shall now enter upon some more details. The magnetic separation of lines in a *non-uniform* field has been treated on a former occasion.[1] The results then obtained and our present observations may be of some interest in connection with certain phenomena observed by Hale. We intend to return to this subject. It seems of interest to allude presently to W. M. Mitchell's descriptions of the various types of spot lines as indicated in the diagram published in the *Astrophysical Journal*, **22**, 6, 1905, and copied with some modifications in the *Transactions of the International Solar Union*.[2]

Our Plate XVIII, Fig. 14, has been copied from Mitchell's paper. The types *a*, *b*, *d*, and *e* of the figure are very characteristic. Type *g* perhaps falls under the type of lines invisible without nicol, mentioned in 7 above. In Fig. 15 are represented some separations observed in the laboratory *without* nicol or other analyzer; a', b', e' have been taken in non-uniform fields. b' is the quartet of D_1 observed across the field; a' the sextet of D_2 observed axially in a non-uniform field, very strong in the central part; e' also refers to D_2 in a weaker field, the observation being made across the lines of force. The type d' refers to the D_2 line, when observed in a direction parallel to the field. The field is uniform. The separation gives an example of the superposition phenomenon mentioned in paragraph 7.

[1] Zeeman, *Proceedings of the Amsterdam Academy*, April 1906, November 1907.

[2] *Transactions of the International Union for Solar Research*, **2**, 199, 1908.

The analogy of the type d', Fig. 15, and the type of the "winged line" seems very remarkable. Of course observation of the state of polarization would be necessary in order to prove the analogy.

20. Explanation of Plates XVI–XVIII. Figs. 1–13 are about thirteen-fold enlargements of the images given by the grating of the absorption lines D_1 and D_2 in a magnetic field.

The upper and lower of the three strips of these figures relate to (oppositely) polarized light; in the central strip the phenomenon is represented as it is seen in natural light.

Plate XVI, Figs. 1, 2, 3, 4, observations perpendicular to lines of force with different vapor-density. Figs. 5, 6, observations parallel to lines of force with different vapor-density.

Plate XVII, Figs. 7, 8, observation at $\theta=60°$, calc-spar rhomb alone. Fig. 9, $\theta=60°$, calc-spar combined with Fresnel rhomb. Figs. 10, 11, $\theta=45°$. Fig. 12, $\theta=30°$.

Plate XVIII, Fig. 13, $\theta=39°$. Fig. 14, types of sun-spot lines (adopted from Mitchell). Figs. 15, a', b', e', separations in non-uniform laboratory fields; d' superposition phenomenon, in paragraph 7.

21. The outer components of a magnetically divided line, if observed in a direction inclined to the lines of force under an angle θ, are elliptically polarized. In our experiments of paragraphs 12–17 we frequently referred to this elliptical polarization. In paragraph 12 the simple rules were summarized which relate to the ellipses characterizing the state of polarization of the outer components, if *very narrow* spectral lines are observed in a *strong* field. The linear vibrations of the central component of a triplet lie, according to the elementary theory, in the plane passing through the ray and a line of force, and the amplitude is proportional to $\sin\theta$. Righi's theoretical considerations in his paper cited in paragraph 1 also agree with this conclusion.

22. In Voigt's[1] theoretical investigation of the magnetic effect in a direction inclined to the lines of force, the remarkable conclusion is drawn that the central component also of a triplet may execute an elliptical vibration. This result is most closely connected

[1] "Weiteres zur Theorie der magneto-optischen Wirkungen," *Annalen der Physik*, 1, 389, 1900.

with the consideration of the mutual action between neighboring molecules.

Lorentz' considerations concerning our present subject (see paragraph 1 above) give results which we may be permitted to summarize here briefly. For arbitrarily chosen values of the angle θ between the ray and the magnetic force for every frequency two elliptical vibrations of opposite directions can be transmitted. In the case of the *outer* components of a sharp triplet one of the two elliptic vibrations is absorbed. If we are not dealing with a sharp triplet, i.e., three absorption bands that are completely separated, we can still say something about the vibration-ellipses of the outer components. Let axes OY and OX' be chosen, the one normal to the plane passing through the ray and the magnetic force, the other perpendicular to the ray and lying in the plane just mentioned. Then one of the characteristic vibration-ellipses can be considered as the reflected image of the other with respect to a line bisecting the angle $X'OY$. This rule also applies to the direction of motion in the two ellipses.

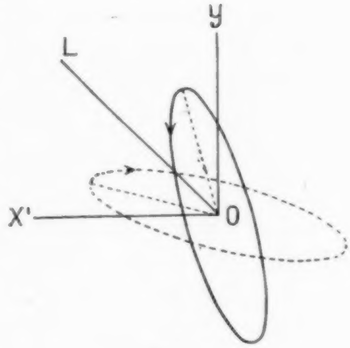


FIG. 5

The nature of the phenomena that will be observed for rays of a frequency corresponding to the *central* line of the triplet depends upon the value of θ being greater or smaller than a certain angle θ_1. This latter is determined by the equation

$$\tan\theta_1 \sin\theta_1 = \frac{g}{\nu}.$$

The quantity g may be regarded as a measure of the width of an absorption line and depends upon the constants of the vapor; ν is determined by the change of the frequency of the free vibrations of the electrons and has a value proportional to the strength of the field.

If $\theta > \theta_1$, then two linearly polarized beams with equal indices

of refraction and different absorptive indices can be propagated. The rectilinear vibrations make equal angles with the line *OL*, bisecting the angle *X'OY*. The absorption is stronger for the beam whose vibrations make the smaller angle with the direction of the field. In the figure the more strongly absorbed vibration is indicated by a thicker arrow. As θ decreases, the vibrations of the two principal beams approach more and more to *OL*, so that for $\theta=\theta_1$ both directions coincide with the bisectrix. The two principal beams are now equally absorbed also.

When $\theta<\theta_1$, the state of things is wholly different. In this case two elliptically polarized beams can be propagated; they are equally absorbed, but have different velocities of propagation. For both beams the characteristic ellipses are the same, but described in opposite directions. One of the axes of the ellipses coincides with the line *OL* in Fig. 6. The ellipses become less and less eccentric as the wave becomes less inclined to the direction of the field. For $\theta=0$ the ellipses become circles described in opposite directions. A further approximation for $\theta=\theta_1$ shows that in this case the two vibrations do not coincide exactly. As in the general case, there are two distinct beams with different characteristic ellipses, both deviating somewhat from the line *OL* of Fig. 6. The regions of the longitudinal and the transverse magnetic effect overlap to a certain extent and are not sharply separated from each other at the angle θ_1.

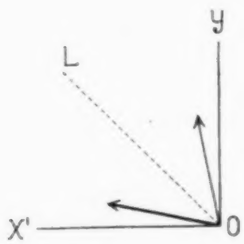


FIG. 6

23. There are three results of Lorentz' theory that probably admit of experimental verification. Let us imagine the absorbing vapor placed in such circumstances that the elementary theory cannot be applied. The components of a divided line are now not neatly separated by practically transparent regions. The vapor-density must be chosen relatively great and the magnetic intensity rather small. As always in the present paper, we suppose the lines of force to be horizontal; we examine the propagation of the light also in a horizontal plane.

The three predictions referred to, which apply, if we exclude the cases of the true longitudinal and transverse effects, are: (1) the major axes of the vibration-ellipses of the outer components deviate from the vertical line; (2) the vibrations of the middle component (c.q. components) are, depending on circumstances, either linear and not horizontal, or elliptic, the axes of the ellipse being inclined to the horizon; (3) there exists an angle θ_1 separating the regions of the longitudinal and the transverse magnetic effect.

OBLIQUE POSITION OF THE VIBRATION-ELLIPSES OF THE OUTER COMPONENTS

24. We succeeded in establishing experimentally the oblique position of the vibration-ellipses in the inverse magnetic effect of the D lines; the amount of the slope of the axes we could measure. The obliquity is far from striking. When θ was already such that the ellipticity was very marked, it was only after some difficulty that we could make sure of the obliquity. Some details of a definite case may be given. With $\theta = 69°$ and a field of about 18,000 gausses the first observations were made. Attention was given to D_2, the vapor-density being regulated so that the outer components of the sextet could not be seen separately. Before the slit of the spectroscope a nicol was placed with its plane of vibration at an azimuth of say 35° with the horizon. The central part of the resolution figure is now very dark; the outer components of the pseudo-triplet, however, are only faintly visible. This has the advantage of increasing the visibility of small changes of the intensity of the outer components.

The direction of the field we denote as field-direction 1. *With the reversed field-direction 2, the outer components became darker.* This experiment was repeated several times with the same result. The nicol then was placed in a position symmetrical to the one just mentioned. Now with field-direction 1, the outer components were darker. From these experiments we must conclude that a vertical line is not an axis of symmetry of the vibration-ellipses of the outer components, hence that the position of these ellipses is oblique.

25. The direction of the smaller axis of the vibration-ellipse we

measured for $\theta=69°$, the vapor-density being between the first and second phase (13). In front of the slit of the spectroscope was introduced a nicol, mounted upon a divided circle which gives the rotation of the nicol in degrees. The vanishing or reappearing of the outer components gave a good criterion for the determination of the smaller axis and therefore of the major axis of the vibration-ellipse. The measurements gave the result that under the circumstances of the experiment the major axis made an angle of 5 degrees with the vertical. The obliquity was the same in amount and direction for the components toward the red and toward the violet. The diagram, Fig. 7, illustrates the relation between the slope of the ellipses and the direction of the field.

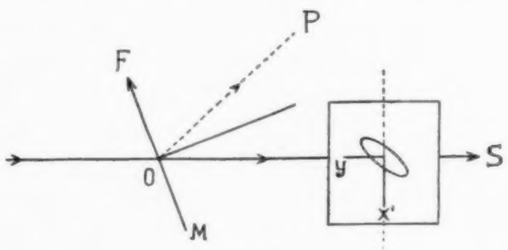


FIG. 7

Let *OS* be the beam, which traverses the source of light placed in *O*, and *OF* the direction of the magnetic force. For an observer looking in the direction *SO*, the upper part of the vibration-ellipse is inclined toward the right. The plane YX', containing the ellipse, is normal to the ray and in the figure has been rotated round the dotted line until brought into coincidence with the plane *SOM*. That side of the plane which was visible from *S* can now be seen. Both the ellipse toward the red, and the ellipse described in the opposite direction toward the violet, have the same slope with a given direction of the magnetic field, as was remarked above.

26. *The same* sodium flame, investigated as to the inverse effect in the direction *OS*, we studied in the direction *OP* (i.e., for an angle $FOP=MOS=180°-\theta$) for the phenomenon of partial polarization, discovered by Egoroff and Georgiewsky. A small telescope focused upon the flame was used and provided with a Savart plate

and a nicol. This polariscope is mounted upon a divided circle graduated in degrees. The direction in which the fringes were most brilliant was determined in order to detect a possible deviation of the plane of maximum polarization from the vertical. It was easily seen that there was such a deviation. The fringes were most clear if for the observer in P their direction was from the upper left to the lower right quadrant, the direction of the field being always as indicated in the figure. After reversal of the magnetic field the fringes became indistinct. They became distinct again if the principal direction of the polariscope was from the upper right to the lower left quadrant. The result of these observations at least proves that the whole phenomenon is asymmetrical with respect to the vertical, and hence proves the presence of oblique vibrations. In a conversation with one of the authors Professor Lorentz had kindly communicated that he observed phenomena of the kind described in this paragraph.

27. In the experiment of the last paragraph the axis of the telescope must be placed carefully in a horizontal plane passing through the poles of the electromagnet. If the observation is made in a plane which is not horizontal, an apparent slope of the axes of the vibration-ellipses becomes manifest, as is easily seen from a geometrical consideration.

28. The position of the plane of maximum polarization can be determined rather accurately. The obliquity of the major axis of the outer ellipses of sextet and quartet in one experiment was 5°; with the very same vapor-density and the same strength of field, the plane of partial polarization made an agle of 21° with the vertical. At first sight it seems rather startling that the polariscope of Savart is so sensitive to the obliquity of the ellipses.

The phenomenon of the partial polarization of the emitted light is very complicated and the complete theory still outstanding. It does not seem doubtful, however, in what direction we have to look for the explanation of the remarkable difference between the indications of the two instruments. They measure different quantities.

As long as the inclination of the vibration-ellipses of the emitted light is zero, the total light also vibrates symmetrically relatively

to the vertical. If the inclination is not zero, however, but has the value α, the plane of maximum resultant luminous motion is inclined at an angle $\alpha+\beta$, which may be occasionally much greater.

The light emitted by the sodium flame contains: (1) horizontal vibrations of intensity c^2 (we neglect here a change mentioned in paragraph 30 below); (2) elliptic vibrations, the major axes of which form an angle α with the vertical. Let the principal axes of these ellipses be a and b.

The intensity I_x in a direction OX becomes

$$I_x=c^2 \sin^2 (\alpha+\beta)+a^2 \cos^2 \beta+b^2 \sin^2 \beta. \tag{1}$$

This expression becomes a maximum for a value of β satisfying

$$c^2 \sin 2(\alpha+\beta)+(b^2-a^2) \sin 2\beta=0. \tag{2}$$

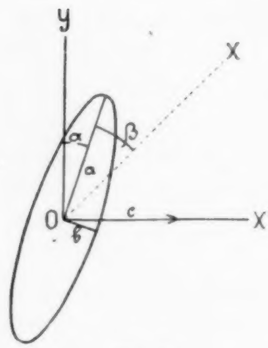


FIG. 8

Hence it follows at once that β cannot be zero, for otherwise α ought to be zero also.

From (2) we obtain

$$\frac{\sin 2(\alpha+\beta)}{\sin 2\beta}=\frac{a^2-b^2}{c^2}. \tag{3}$$

Hence the value of β depends upon the intensities of the horizontal and vertical vibrations. We always have $a>b$; in the emitted light the vertical vibrations generally preponderate, hence also $a>c$. We conclude that β can be positive only.

If we make $a=5°$, $a+\beta=21°$, $b=0.3a$ (see 29), equation (3) gives

$$\frac{a^2}{c^2}=1.4:$$

This is a plausible value. Hence there is no contradiction between the observations made with the polariscope and the results obtained with the nicol alone.

29. We made, with the inverse effect, some measurements of the ellipticity of the outer components at different angles of incidence. We used for this investigation the well-known method of

the quarter-wave plate and nicol. The axes of the quarter-wave plate being placed parallel to the axes of the original ellipse, the resulting light is plane polarized. Let b and a be the horizontal and vertical or the nearly horizontal and the nearly vertical axes, then

$$\frac{b}{a}=\tan \alpha .$$

The mica quarter-plate used proved to be very accurate for light of the refrangibility of the sodium lines, when tried by the method described on a former occasion.[1] Three determinations gave for the deviation from an exact quarter-wave plate the values 1.8, 0.1, 1.0 per cent. For our present determinations this accuracy of the plate is quite superfluous. The measurements are very difficult, relating as they do to the mean of the outer components of the sextet, hence to an extremely narrow part of the spectrum. Moreover the density of the vapor can be defined only approximately (10).

The following table embodies the results concerning the ellipticity of the outer components of the sextet obtained in a somewhat extended series of measurements.

θ	b/a		Remarks
69°5	0.31 0.31 0.28	0.30	Vapor of intermediate density (§ 10)
47°	0.45 0.45	0.45	Vapor of intermediate density (§ 10)
	0.47 0.50	0.47	Vapor somewhat denser
39°	0.67 0.70 0.70 0.60 0.64 0.67 0.63 0.65 0.65	0.66	Very dilute vapor (§ 10)

[1] Zeeman, *Amsterdam Proceedings*, October 30, 1909.

The ratio of the axes at a certain angle undoubtedly somewhat depends upon the vapor-density. Part of the oscillations of the results obtained at the same angle must be ascribed to this cause.

At $\theta = 69^\circ.5$ and with dense vapor the inclination of the major axis of the ellipse was 6°; with very dilute vapor the value zero was obtained. At $\theta = 47^\circ$ and with vapor of intermediate density the inclination was $4^\circ.5$. The Savart fringes then made an angle of 28° with the vertical.

OBLIQUE POSITION OF THE VIBRATIONS OF THE MIDDLE COMPONENTS

30. Whereas the inclination of the vibration-ellipses of the outer components could be demonstrated first for the sextet, it was for the quartet, on the contrary, that we first succeeded in verifying the second of Lorentz' above-mentioned conclusions (23). The deviation of the vibrations of the middle components of the quartet from the horizontal line can be shown in the same manner as the inclination of the ellipses (24).

The principal section of the nicol before the slit was placed at an angle of about 30° with the horizon. The outer components of the quartet of D_1 are then hardly visible. The inner components are rather dark. The direction of the field is indicated as direction 1. Under the influence of the reverse field 2, the middle components become more black. If the nicol be placed in the symmetrical position, then it is with the field-direction 1 that the middle components are most distinct. The angle θ in this experiment was 47°.

Two different attempts to measure the angle between the vibration and the horizontal gave the results $4^\circ.5$, and $5^\circ.5$. These measurements are very difficult, however, and perhaps indicate only the order of magnitude of the inclination. The vicinity of the outer components largely interferes with the accuracy of the adjustment of the nicol, for while it is moved about near the position of extinction and approaches to a vertical direction the greater intensity of the outer components distracts the eye.

31. We have made yet another experiment which confirms the result of paragraph 30 for both the sodium lines and also exhibits

the relation between the inclinations of the different components. This connection is shown diagrammatically for a triplet in Fig. 9; for the result obtained with the middle components of the quartet and the sextet certainly can be applied qualitatively to the triplet.

The experiment was the following: the principal section of the nicol made an angle of $+40^\circ$ with the vertical; the positive direction in Fig. 9 was counter-clockwise. Then the nicol was placed at 320° (i.e., in the symmetrical position). The last position may be indicated as position *B*, the first mentioned as position *A*. The direction of the field remains unchanged. In position *A* all lines were weaker than in position *B*.

Hence we conclude that the ellipses as well as the vibrations of the middle components are inclined; moreover, that the relative position of the vibrations must be that shown in Fig. 9.

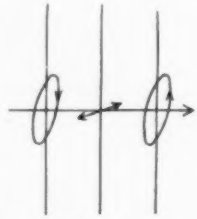


FIG. 9

32. In the important paper already frequently mentioned (note p. 291 of the paper cited in paragraph 1 above) Righi says that Voigt's theoretical investigation of the general case of propagation of light in a direction inclined to the lines of force was published too late to guide him in his investigation. Righi expresses the opinion that it is rather improbable that in the course of his numerous observations peculiarities in the behavior of the middle components as indicated by Voigt could have escaped him, and that Lorentz' elementary theory is in accordance with all the observed phenomena.

This seems in contradiction with our experiments. This contradiction vanishes, however, if we assume that the vapor in Righi's experiments was very dilute, or the field so intense that the components were neatly separated. Under such circumstances our observations are also in complete accordance with the elementary theory, at least as to the polarization of the components and the direction of the vibrations. Neither was it in Righi's experiments a matter of course to reverse the direction of the magnetic field, the procedure which most easily exhibits any obliquity of the vibrations.

APPLICATION OF THE RESULTS OF PARAGRAPHS 24–31 TO THE INTERPRETATION OF SUN-SPOT SPECTRA

33. The vibrations of the middle component of a triplet are parallel to the lines of force. The outer components vibrate linearly at right angles to the field. These rules also apply to dense vapors, if only the pure transverse magnetic effect be under consideration. If we assume that the direction of observation is oblique to the lines of force, then only in the case of very dilute vapors can the projection of the magnetic force on a plane normal to the line of vision be found according to the rules of the elementary theory from the direction of the vibrations. If, however, the components of an inverse triplet are not neatly separated by practically transparent parts—and the sun-spot lines seem to belong to this class of lines—the particulars diagrammatically illustrated by Fig. 9 are to be taken into consideration.

In drawing charts of the magnetic fields in sun-spots, showing the intensity, the direction, and the polarity of the magnetic force, the determination of the direction of the force will give some difficulties. The value of the correction to the indications of the elementary theory necessary in some cases will be given on another occasion.

The rule which determines the direction of the deviation may be indicated here. The direction of rotation in the vibration-ellipses of the outer components toward the red and toward the violet shows whether θ is acute or obtuse. If θ is obtuse (Fig. 7), then the relative position of the directions of the magnetic force, of the major axis of the vibration-ellipses, and of the vibration of the middle component is shown in Fig. 9.

From any point O draw a line OB parallel to the major axis of the vibration-ellipses of the outer components and a line OM parallel to the vibration of the middle component, the angle BOM being always chosen acute. The projection OF of the magnetic force on a plane normal to the line of sight then makes a positive acute angle with OB, the angle BOF being greater than BOM, the positive direction being reckoned from OB to OM.

By ascertaining whether or not the major axes of the ellipses and the vibrations of the middle component are perpendicular to

each other, we can make sure whether the elementary theory may be applied or not.

DEMONSTRATION OF OBLIQUE POSITION OF VIBRATIONS BY MEANS OF HALF-WAVE-LENGTH PLATE

34. The observations in the preceding portion of our communication relate to the region between $\theta = 90°$ and $\theta = 39°$, including the two principal directions. We now intend to describe experiments relative to the remaining region between $\theta = 39°$ and $0°$.

This region seemed very interesting because under suitably chosen circumstances it probably would contain the angle θ_1 of Lorentz, separating the regions of the longitudinal and the transverse magnetic effect. The principal object we had in view in undertaking this third part of our investigation was to prove experimentally the existence of an angle of the kind mentioned. We think that we attained our purpose.

Before proceeding to describe these experiments, we shall mention a method for verifying the results (paragraphs 24–32) relating to the oblique position of the vibration-ellipses of the outer components and that of the vibrations of the inner components, but without commutation of the current in the electromagnet.

Whereas in our former experiments the *difference* of the intensity of the components by *commutation* of the current gives the proof for the obliquity of the components, the half-wave-length plate demonstrates it at once.

A half-wave-length plate, with one of its principal directions situated horizontally and limited by a horizontal line, is placed near the source of light. Vibrations from the source, making a definite angle with the edge of the plate, after traversing it are rotated through twice that angle. The plate covers only half of the field of view. The directions of the emergent vibrations make the same angles with the horizontal edge as at first, but upon the farther side. An image of the edge is focused upon the slit of the spectroscope; before the slit a nicol is placed.

In one of our experiments, θ being $39°$, the plane of vibration of the nicol was at an angle of $35°$ with the horizon. The magnetic components are now seen unequally dark in the two halves of the

field of view. It appeared possible to photograph the phenomenon; small variations of vapor-density, which may possibly introduce errors with other methods of observation, are now without influence. Reversal of the direction of the current changes the sign of the difference of intensity of the two halves of the field of view.

CONNECTION BETWEEN THE INCLINATION OF THE ELLIPSES IN PARTICULAR CASES

35. The direction of the magnetic field, and that of propagation of the beam traversing the magnetized source of light determine the sense of the inclination of the vibration-ellipses (paragraph 25). If the direction of the field be reversed, the sign of the inclination of the vibration-ellipses also changes. In Fig. 7 (paragraph 25) the connection established by our experiments between the three mentioned directions is given.

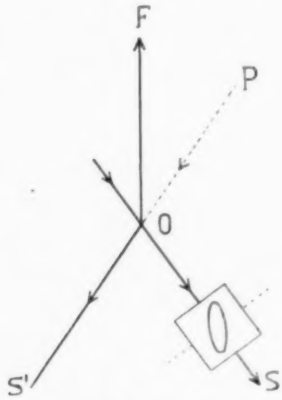


FIG. 10

Let *OF* be the magnetic force, and let the beam, traversing the magnetized flame *O*, be propagated in the direction from *O* to *S*. The inclination of the ellipses in this case is indicated in Fig. 10. The plane normal to the ray and containing the ellipse has been rotated round the dotted line until brought into coincidence with the plane of the paper.

What is the inclination, if the source of light be traversed by the beam in the direction *OS'*?

This question is easily answered by applying the well-known method of reflected images. The geometrical outlines of all things composing a given system, together with the physical processes in the system, which we suppose may be all represented by geometrical figures, we imagine reflected at every instant in a plane *V*. The new system obtained by reflection, which we call the image of the original system, is a possible one, as soon as the last-mentioned one has an objective existence.

Applying this to our experiment (Fig. 11) and placing the plane

V parallel to *OF* and perpendicular to the plane of the paper, we obtain from system I the system II.

The magnetic field in the second system is the inverted image of the field in the first one; indeed, before taking the image of the field we have to substitute it by the equivalent ampere currents. Hence in II the arrow *F′O′* is drawn from *F′* to *O′*. The field in system II being afterward reversed, the inclination of the ellipse

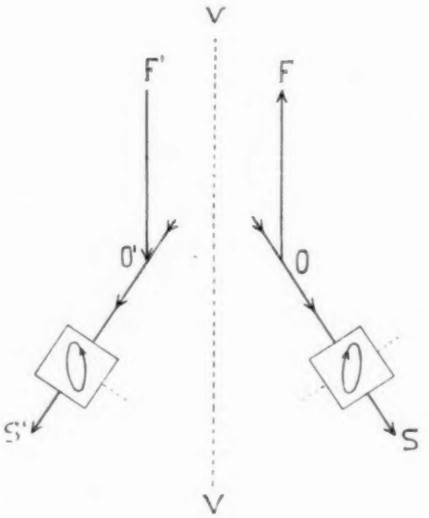


Fig. 11

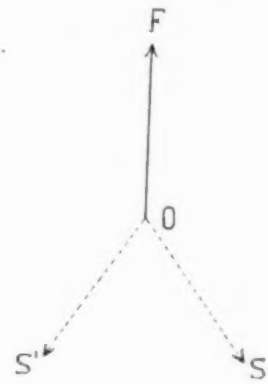


Fig. 12

changes its sign. Hence we conclude that (Fig. 12), if *OF* be the direction of the magnetic field, the inclination of the major axes of the ellipses, as observed from *S* as well as from *S′*, is always from the lower left to the upper right quadrant.

By means of Savart's polariscope all this could be experimentally verified. We come to the same conclusion by using the experimental result of paragraph 26, concerning the inclination of the ellipses in the beam emitted in the direction *OP* (see Fig. 10).

The close connection existing between emission and absorption enables us to predict the phenomena to be seen if light traverses the source in the direction *OS′* (see paragraph 44).

INVESTIGATION CONCERNING THE EXISTENCE OF AN ANGLE θ_1 (36–46)

36. It seems possible to give by different ways experimental proof of the existence of an angle θ_1, separating the regions of the longitudinal and the transverse effect.

The most direct proof would be given if, with a chosen magnetic force, the vapor-density could be changed in such a degree that at last the direction of the vibrations in the issuing beam were inclined at an angle of 45° with the vertical. Then one would observe at the angle θ_1 itself the values of density (width) and magnetic intensity corresponding. The execution of this plan gives rise, however, to serious difficulties.

The significance and the distinctness of the angle θ_1 become manifest also, however, if it be possible to establish the existence of the characteristic phenomena only observable for a direction of observation which forms an angle with the lines of force lying between 0° and θ_1. We have experimentally verified the theoretical inference. We made many experiments belonging to each of the two classes of experiments mentioned and intend to give a few examples of each.

37. *Observations at* $\theta=32°$.—Soft iron cones with a vertex semi-angle of 32° were made and adapted to a Du Bois electromagnet. The intensity of the magnetic field proved sufficient to establish the character of the resolution in the first-order spectrum of the large Rowland grating.

The middle components were especially watched. It is easily established that the vibrations of these components deviate from the horizontal. In order to demonstrate an inclination of 45°, a quartz plate, cut perpendicularly to the axis, and exactly 2 mm thick, was introduced in the beam. This plate rotates the plane of polarization for sodium light $2\times21.7=43°.4$. Vibrations at azimuth 45°, after traversing the plate, become either horizontal or vertical.

Between the plate and the spectroscope slit a calc-spar rhomb was inserted and a horizontal slit placed near the source; two contiguous horizontal images of this slit are now formed on the slit. The one contains the vertical, the other the horizontal constituents

of the beam. The middle components, which at the angle θ under consideration are rather weak, are dependent upon the direction of the current, and are visible either only in the upper or only in the lower of the two strips, if the vapor-density be properly chosen.

This experiment does not prove definitely, however, that the middle components may vibrate under an angle of 45° with the vertical. The rather limited sensitiveness of the method must be taken into account. The experiment certainly proves that the vibrations are inclined relatively to the horizon, at an angle of perhaps 20° or 30°.

It is shown by an observation with the calc-spar rhomb alone, after removal of the quartz plate, that the vibrations are not performed under 45°. A difference between the upper and the lower image is now manifest. This would be impossible if the inclination of the vibrations were 45°. The difference of intensity in the two strips decreases with increased density of the vapor.

All experiments undertaken in order to measure more accurately the inclination gave no decisive results. The weak intensity of the middle components, the feeble separation (to be expected for the observations in view, according to the theory), the perturbation by the vicinity of the outer components, and also the fact that the vibrations become probably slightly elliptic, account for the difficulty of the measurements.

We also investigated the emitted light without the aid of the spectroscope, with a Savart polariscope alone; the emitted light appeared to be nearly unpolarized. The fringes in the polariscope were very weak. This is clearly due to the light containing equal portions of right-handed and left-handed nearly circularly polarized light; the intensity of the light of the middle components is relatively very small and therefore scarcely perceptible in the resulting total intensity. The indistinctness of the fringes made only inaccurate determinations of the position of the plane of polarization possible. An inclination of 42° relatively to the vertical was found.

38. The method of the non-uniform field[1] seemed to open the possibility of a direct reading of the field-intensity corresponding to θ_1, the vapor-density (i.e., the width of the spectral line) being

[1] Zeeman, *Amsterdam Proceedings*, April 1906, November 1907.

given. At $\theta=39°$, a diminished image of the cones of the electromagnet was focused upon the slit-plate of the spectroscope. The magnetic separation is different at different heights, and in the spectroscope the spindle-shaped resolution figure, a photograph of which was given on a former occasion, is seen; but now, as the inverse effect is under consideration, rather dark lines on a luminous background are seen. A nicol with its plane of vibration at 45° with the horizontal is placed before the slit. If the vibrations occur at 45° somewhere in the divided lines, the components must become black at such a place. Width and field-intensity, belonging to the part mentioned of the components, correspond to a value of θ_1 equal to 39°.

No clear result was obtained by means of this method, however, which was tried with several vapor-densities. The change of the state of polarization in the resolution figure apparently is too gradual to prove the existence of θ_1 by direct observation. Our experiments following (paragraphs 39–46) seem, indeed, to leave no doubt as to the real existence of such an angle.

39. In order to extend observations to still smaller angles θ, the second-order spectrum of the large Rowland grating was employed for all following observations. The brightness is still amply sufficient and more details are seen. Even with cones with a vertex semi-angle of 26° the characteristic phenomena may now be advantageously observed. With vapor of intermediate density (paragraph 10) only the outer components of the quartet and sextet are now visible, the phenomenon closely resembling the purely longitudinal one. Middle components make their appearance only after the density is largely increased. The nature of these components appears (paragraph 40), however, to have changed, as is proved by an examination of their state of polarization.

The latter is more easily ascertained, if the components are more widely separated. This is the case in the experiments described in the next paragraphs and therefore we prefer to give some details of the observations made with the more efficient arrangement.

40. A still smaller angle between the directions of the beam and of the field may be employed, and moreover wider separation obtained than in paragraph 39, by looking through axial holes and

deviating the beam in the field by means of two small prisms. A remark of Professor Wertheim Salomonson induced us to give prisms a trial.

The arrangement for $\theta = 16°$ is shown in the figure (Fig. 13). The prisms are fixed to copper tubes, which are put into the bored cones of a Du Bois electromagnet and may be turned about their axes. It is therefore possible to adjust the parallelism of the planes of prisms and to arrange the edges vertically.

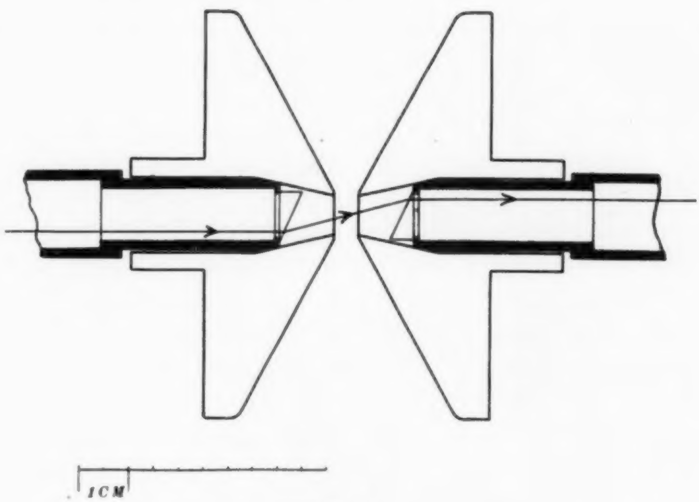


FIG. 13

A drawback inherent to this method is that after some time the interior surfaces of the prisms become covered with some white precipitate. With very dense vapors this inconvenience is rather troublesome. Immediately after introduction of the flame into the interferrum, aqueous vapor condenses upon the prism faces, soon disappearing, however, when the temperature of the prisms has increased. In order to avoid the danger of cracking, the prisms have been placed at some distance from terminal planes of the cones.

Even with very dense vapor (third phase of 10), the field being of the order of 20,000 gausses, the phenomenon closely resembles the purely longitudinal one. No trace of middle components is visible. After an increase, however, of the vapor-density to the limit

obtainable by the introduction of a glass rod, charged with melted salt, into the gas-oxygen flame, two new black lines appeared in the vicinity of D_1; they were clearly visible against the rather dark background formed by the broadened outer components.

These new lines, which have the same period as the middle components, are unpolarized (see paragraphs 41–44).

41. We have come to this conclusion after trying in vain to detect any trace of polarization phenomena of the new components. In the first place, rotation of a nicol placed before the slit of the spectroscope gave no change of intensity of the lines; only the background formed by the nearly, but not accurately, circularly polarized outer components was slightly changed.

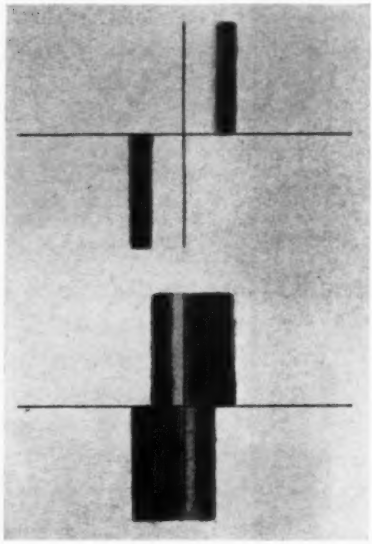


FIG. 14

42. After removal of the nicol a quarter-wave plate with its principal direction at 45° was inserted in the beam and a broad horizontal slit placed near the field. By means of a calc-spar rhomb two strips are obtained, separating the oppositely polarized circular vibrations.

With vapor of intermediate density Fig. 14*A* gives the appearance for D_1. The vertical line represents the reversed line due to the arc light.

With very dense vapor, we get the phenomenon represented in Fig. 14*B*. New components appear in the initially bright parts of the field of view.

The positions of the new components correspond to those of the inner components of the quartet, at least as far as can be judged by eye observation. This observation is confirmed by measurements made on a photograph, it must be said, of only moderate quality.

As to the polarization of the new lines a few remarks may be

made. From an inspection of Fig. 14*B* alone, one might infer a circular polarization of the inner components, of a sign opposite to that of the outer ones. One might be tempted to infer that, under the circumstances of the experiments, the inner components are due to the motion of positive charges. There is no need of discussing the degree of probability of such a conclusion, as it is refuted by the next observation.

42. If the quarter-wave plate be rotated in its own plane so that the principal direction more and more approaches to the horizontal position, the intensity of the outer components decreases. The inner components, which at first are invisible in two of the quadrants, being entirely hidden by the black, broad outer components, are very soon seen as continuous bands crossing at right angles the horizontal separation line. Finally, when the principal direction of the quarter-wave plate has become horizontal, there is, as far as concerns the inner components, no difference at all between the upper and lower fields, and only a slight one as far as concerns the outer components.

43. From the observations recorded in paragraphs 41 and 42 we cannot but conclude that under the circumstances of the experiment the inner components of the *new* quartet are unpolarized. This result seems paradoxical, because one now has become accustomed to expect polarization of all magnetically separated and displaced lines. The result, however, seems to be in perfect accordance with theory, at least if it be permitted to apply to the middle components of the quartet, the theoretical inference drawn for the central component of the triplet.

Lorentz has proved that in the case of a triplet for a frequency $n=n_0$ and $\theta<\theta_1$ two oppositely elliptically polarized beams may be transmitted, having the same index of absorption, but unequal velocities of propagation. The characteristic vibration-ellipses for the two beams are the same, but described in opposite directions (see also 22 above.) Since the indices of absorption of the two beams are equal, we may expect that, under the circumstances mentioned, a magnetized vapor can produce in a continuous, unpolarized spectrum, unpolarized absorption lines only.

44. The consideration in paragraph 35 of the reflected image of

a system was made in order to show that the inclination of the ellipses remains unaffected by a change of sign of the angle between the line of force and the ray.

45. *Quartet for* $\theta=0$.—By increasing still further the vapor-density necessary for the experiment of paragraph 43, we were able to observe, even in the direction $\theta=0$, the two unpolarized lines, corresponding to the inner components of the quartet. The outer components, however, have then become extremely diffuse. It is certainly remarkable that the two new components are still relatively narrow. The theoretical reason for this feature of the phenomenon has still to be worked out.

It is, however, in accordance with theory (always on the supposition that it does apply directly to the quartet) that for $\theta=0°$ the density of the vapor must exceed that for $\theta=16°$, in order to render visible the new lines. Indeed, according to the formulae (42) and (26) of Lorentz' paper (cited in 1 above) the absorptive index decreases with decreasing θ.

The experiments (39–43) seem to give conclusive evidence that an angle θ_1 really exists. Indeed, phenomena of the kind described in the last paragraphs are to be expected in a region only between θ_1 and 0°.

The experimental verification of Lorentz' deductions, formulated in 23 above, gives a new proof of the rational connection established by Voigt's theory of the inverse magnetic effect between diverse phenomena.

A more accurate measurement of θ_1, the vapor-density and the field being chosen, must be postponed.

46. The new type of magnetic separation, with some components polarized, the other ones unpolarized, which returns to the ordinary separation by decrease of vapor-density, we were able to observe also with D_2. Since the density of the vapor must be great in the present experiment, the effects observed with D_2, which splits up into a pseudo-triplet, are less clear and characteristic than with D_1. We therefore restricted the detailed description of our observations to the case of D_1.

PHYSICAL INSTITUTE
THE UNIVERSITY OF AMSTERDAM
1910

MASS-RATIOS IN THE SYSTEMS OF *KRÜGER 60* AND *CASTOR*

By HENRY NORRIS RUSSELL

I. *Krüger 60*

Burnham's General Catalogue of Double Stars, 11761 ($\alpha = 22^h\ 24^m.5$, $\delta = +57°\ 13'$, 1900)

In Professor Barnard's thorough discussion of the parallax and proper motion of this star,[1] he points out that the motion of the principal component, A ($9^m.7$), with respect to the distant optical companion C ($10^m.2$), is distinctly curved, showing that the faint component B ($11^m.0$) of the binary system has a mass comparable with that of A.

These observations alone would give a fairly good determination of the mass-ratio. The addition of unpublished measures of the last two years, which Professor Barnard has very kindly communicated to the writer, makes it possible to fix this quantity within quite narrow limits.

If the distance and position angle of C, referred to A, are s, p, and those of B are s', p', their apparent rectangular co-ordinates, referred to axes whose position angles are p_0 and $p_0+90°$, will be

$$x = s\ \cos\ (p-p_0),\ x' = s'\ \cos\ (p'-p_0),$$
$$y = s\ \sin\ (p-p_0),\ y' = s'\ \sin\ (p'-p_0).$$

If k is the ratio of the mass of B to that of A and B combined, the co-ordinates of the center of gravity of the binary system will be kx', ky', and, since the motion of C relative to this point must be uniform, we must have

$$x = a+b(t-t_0)+kx',\ y = a'+b'(t-t_0)+ky',$$

where t_0 is any convenient epoch. Each set of simultaneous observations of AB and AC gives us an equation of condition for the constants a, b, k, and another for a', b', k.

No knowledge of the period of the binary system, or of the form of the complete orbit, is necessary in forming these equations, which depend only on the relative motion during the interval

[1] *Monthly Notices*, **68**, 629, 1908.

covered by the observations; and it is obvious that the accuracy with which k can be determined depends on the magnitude of the departures of the motion of B (relative to A) from uniformity, during this interval.

The data on which the present discussion is based are as follows. They are taken from Barnard's paper, already mentioned, except for the last two lines which are annual means of the unpublished measures referred to above. The rectangular co-ordinates x and y are computed with $p_0 = 59^\circ.00$.

The parallax of A is $0''.25$ (according to three very accordant determinations by Barnard, Schlesinger, and the writer). The observed co-ordinates of C, relative to A, therefore require to be increased by the corrections

$$\Delta x = \Delta s = +0''.25 \cos(\odot - 93^\circ),$$
$$\Delta y = \qquad +0''.22 \cos(\odot - 179^\circ).$$

The tabular values of s are already corrected for parallax (this having been done by Barnard). Those of p are uncorrected, but the correction has been applied to the tabular y's. The mean of the corrections for the individual dates has been taken when necessary. The position angles have also been corrected for precession, to 1900, at the rate of $-0^\circ.0042$ annually.

	AB					AC				
Date	p	s	n	x	y	p'	s'	n	x'	y'
1890.79.....	178°.8	2".32	1	−1".16	+2".02	56°.26	26".73	1	+26".70	−1".48
1898.45.....	140.7	3.19	5	+0.46	3.15	58.70	34.64	5	34.64	0.15
1900.84.....	133.6	3.22	8?	0.86	3.11	59.20	36.36	8?	36.36	+0.02
1901.77.....	130.7	3.28	7	1.03	3.12	59.63	37.23	7	37.23	0.19
1902.76.....	127.0	3.36	4	1.26	3.13	59.65	38.25	4	38.25	0.21
1903.63.....	123.3	3.37	15	1.47	3.04	59.58	39.12	32	39.12	0.30
1904.63.....	119.6	3.38	19	1.66	2.95	59.44	40.03	27	40.03	0.19
1905.91.....	115.3	3.31	2	1.84	2.76	59.66	41.21	2	41.21	0.37
1906.56.....	112.2	3.32	7	2.00	2.66	59.44	41.83	7	41.83	0.21
1907.57.....	107.2	3.28	12	2.18	2.44	59.45	42.84	11	42.84	0.21
1908.36.....	104.4	3.18	6	2.24	2.27	59.11	43.66	8	43.66	0.22
1909.63.....	98.8	3.14	16	2.41	2.01	59.39	44.87	17	44.87	0.25
1910.59.....	94.9	3.13	9	+2.54	+1.84	59.49	45.77	9	+45.77	+0.25

The first measure is by Burnham, the second by Eric Doolittle, and the third, the mean of the results of Doolittle and Barnard.

All the rest are Barnard's. As the latter points out (*op. cit.*, p. 638), it is desirable in such an investigation to use the measures of one observer as much as possible. The number of nights' measures combined into each mean place is denoted by n. In one or two cases the measures of AB have been interpolated to the epoch of those of AC (differing by a month or less).

The motion of B, relative to A, is so nearly uniform in x that only the y's can furnish a reliable value of k. An approximate solution gives

$$y = -1''63 + 0''092(t-1900) + 0.5\,y'.$$

The residuals from this solution are the absolute terms of the equations of condition given below. The unknowns in these are $a = \Delta a' + 2''70 \Delta k$, $\beta = \Delta b'$, $\gamma = \Delta k$, where $\Delta a'$, $\Delta b'$, Δk are the corrections to be added to the approximate values.

		O.−C.	Wt.			O.−C.	Wt.
$a-9.2\beta-0.69\gamma=$	$-0''01$	$-.03$	1	$a+5.9\beta+0.06\gamma=$	$+0''08$	$+.09$	$\frac{1}{2}$
$a-1.5\beta+0.42\gamma=$	$+\ .06$	$+.03$	1	$a+6.6\beta-0.04\gamma=$	$-\ .09$	$-.08$	1
$a+0.8\beta+0.46\gamma=$	$+\ .01$	$-.01$	1	$a+7.6\beta-0.22\gamma=$	$-\ .08$	$-.06$	1
$a+1.8\beta+0.43\gamma=$	$+\ .10$	$+.08$	1	$a+8.4\beta-0.44\gamma=$	$-\ .06$	$-.03$	1
$a+2.8\beta+0.38\gamma=$	$+\ .02$	$+.01$	1	$a+9.6\beta-0.65\gamma=$	$-\ .02$	$+.02$	2
$a+3.6\beta+0.32\gamma=$	$+\ .08$	$+.07$	3	$a+10.6\beta-0.90\gamma=$	$-\ .01$	$+.04$	1
$a+4.6\beta+0.22\gamma=$	$-\ .09$	$-.09$	3				

The resulting normal equations are

$$\begin{aligned} 17.5a + 74.5\beta - 0.25\gamma &= -0''02, \\ +688.\ \beta - 11.9\ \gamma &= -1.91, \\ +\ 3.55\gamma &= +0.200, \end{aligned}$$

whence $a = +0''011$, $\beta = -0''0034$, $\gamma = +0.032$. The probable error of unit weight is $\pm 0''054$, and the final values of a', b', k are

$$\begin{aligned} a' &= -1''70 \pm 0''084, \\ b' &= +0.089 \pm 0.003, \\ k &= +0.531 \pm 0.031. \end{aligned}$$

The relatively large probable error of a' is due to the term $2''70\ \Delta k$ which it contains.

The mass of B appears therefore to be slightly greater than that of the brighter star A, the ratio $\frac{B}{A}$ being $\frac{k}{1-k}$, or $\underline{\underline{\mathbf{1.14 \pm 0.14.}}}$

Comparison with the probable errors of other determinations of similar ratios—such as those given in the appendix to Boss's *Preliminary General Catalogue*—shows that this may be regarded as a fairly good determination; for which reason it is now published, although the observations of ten years more will confine the uncertainty within much narrower limits.

Correcting the x co-ordinates, with this value of k, the annual motion of C, relative to the center of gravity of AB, is found to be $+0\overset{''}{.}861 \pm 0\overset{''}{.}006$.

The proper motion of the system, relative to this star, is therefore $0\overset{''}{.}866 \pm 0\overset{''}{.}006$ in position angle $244\overset{\circ}{.}9 \pm 0\overset{\circ}{.}2$. The orbital motion of the principal star accounts for the difference between this and the value $0\overset{''}{.}968$ in $246\overset{\circ}{.}5$, found by Barnard for the latter for the interval 1890–1905. Both values, however, require unknown corrections for the proper motion of star C, which cannot safely be neglected in dealing with observations of such accuracy.

II. *Castor* (α *Geminorum*)

Though this binary has been under observation for nearly 200 years, the elements of its orbit are still very uncertain. Were they accurately known, the spectroscopic observations of H. D. Curtis[1] would enable us to find very reliable values of the parallax and mass.

It may be shown that the existing uncertainty of the elements falls almost entirely upon the determination of the parallax, and does not seriously influence that of the mass of the system.

Let i be the angle which the relative velocity of the two components in space makes with the plane tangent to the celestial sphere, v the component of this velocity parallel to this plane (in seconds of arc per year), and ρ that perpendicular to it (in astronomical units per year). Then $v = \pi\rho \cot i$, where π is the star's parallax.

If now p is the apparent position angle of the velocity v, as projected on the plane of reference, and Ω and γ are the node and inclination of the orbit-plane, $\tan i = \tan \gamma \sin (p - \Omega)$, and we have

$$\pi = \frac{v}{\rho} \sin (p - \Omega) \tan \gamma. \tag{1}$$

[1] *Astrophysical Journal*, **23**, 351, 1906.

From Curtis' observations, the value of ρ (for the centers of mass of the rapid spectroscopic binaries which form the components of the visual system) is 7.18 ± 0.23 km/sec or 1.52 ± 0.05 astronomical units per year, at the epoch 1904.9. v and p can be found directly from the micrometer measures of the last few years. These give the mean places.[1]

	Position Angle	O.−C.	Distance	O.−C.
1897.88......	226°4	−0°2	5″69	−0″02
1900.46......	225.3	−0.1	5.66	0.00
1903.43......	224.4	+0.1	5.63	+0.03
1906.03......	224.0	+0.8	5.54	−0.01
1908.33......	221.6	−0.7	5.52	+0.02
1909.32......	221.8	0.0	5.45	−0.03

From these with equal weights, we find:

$$p = (224°1 \pm 0°15) - (0°415 \pm 0°037)\ (t - 1904),$$
$$s = (5''59 \pm 0''007) - (0''0200 \pm 0''0017)\ (t - 1904).$$

A check upon the value of $\frac{dp}{dt}$ may be obtained from the principle of the uniform description of areas in the apparent orbit. According to Doberck[2] the projected radius vector swept over 7.658 square seconds of arc between 1832.0 and 1896.0. The corresponding angular velocity at distance 5″59 is −0°438 per annum —agreeing with the observed value within its probable error. The computed value obviously deserves the preference (its probable error being doubtless less than 1 per cent of its value). Combining it with the observed value of $\frac{dr}{dt}$, we find, for 1904:

$$v = 0''047 \pm 0''001$$
$$p = 109° \pm 2°,$$

whence, substituting in (1)

$$\pi \operatorname{cosec}\ (109° - \Omega) \cot \gamma = 0''0313 \pm 0''0013 \qquad (2)$$

in which the probable errors of both ρ and v are taken into account.

[1] The first three are derived from measures given by Lewis, *Memoirs R.A.S.*, **56**, 217, 1908; the others from observations published in the *Astronomische Nachrichten*, for 1905–1910.

[2] *Astronomische Nachrichten*, **166**, 145, 1904.

The numerical terms in this equation depend only on modern spectroscopic and micrometric observations, and are independent of any hypotheses about the orbit; but to find the parallax we must know the position of the orbit-plane.

The extent to which the latter is uncertain is shown by comparing the five orbits recently derived by Doberck (*op. cit.*) and by Lohse.[1] All of these represent the observations (including Bradley's estimated position angle of 1720) fairly well; but the first and last give distinct systematic deviations, in opposite senses, from the observed positions so that the range of uncertainty cannot be much greater than the table indicates.

	P	a	e	Ω	γ	π	M
Lohse I........	249y	7″6	0.80	203°1	73°1	0″103	6.3
Doberck III	268	7.3	0.75	209.5	73.0	0.101	5.3
Lohse II	298	5.9	0.58	207.8	65.4	0.068	7.4
Doberck IV.....	347	5.8	0.44	213.9	63.6	0.061	7.1
Doberck V	502	6.5	0.23	222.6	61.9	0.053	7.3

All the elements vary through a wide range; but if they are plotted as functions of the period, the individual points lie in all cases near a smooth curve. They may therefore be regarded as functions of a single ill-determined quantity, e.g., the period. In other words, there is practically only one way of drawing an ellipse of given area so as to represent the observations, but a great deal of latitude as regards this area.

The values of the parallax and mass, computed for each set of elements with the aid of equation (2),[2] will also be functions of the assumed period. The parallaxes vary through a considerable range (though not more than would at present be considered satisfactory in the case of as many direct determinations). The computed masses are remarkably accordant—the variations in period, major axis, and parallax compensating one another almost perfectly. This would not generally happen, and must be due to

[1] *Publikationen des astrophysikalischen Observatoriums zu Potsdam*, **20**, 92, 1908.

[2] As the apparent direction of motion determined from the micrometer observations has been used in all cases (instead of that of the tangent to the individual apparent orbits), these values will differ slightly from the results of direct computation from the radial velocity and the orbital elements. It appears, to the writer at least, that the present process, involving as it does the minimum of uncertain data, is preferable.

some unusually favorable chance as regards the form and position of the orbit in this particular case; but the fact is put beyond question by the numerical calculations.

We may therefore conclude that the parallax of *Castor* is approximately $0\rlap{.}''08 \pm 0\rlap{.}''03$, and that the mass of the whole system is about 6.5 ± 1.0 times that of the sun.

Little is yet known concerning the ratio of the masses of the components of the visual system. Crommelin has shown[1] that the path of the bright component, derived from meridian observations, is much more curved than that of the fainter, indicating that the latter has the greater mass. The same conclusion may be deduced from the micrometer measures of the distant companion, which may be summarized as follows (setting $p_0 = 163\rlap{.}^\circ5$):

	AC			Differences from Uniform Motion	
	p	$s = x$	y	Δx	Δy
1835.2......	162°5	72″52	−1″27	+0″06	−0″15
1846.0......	162.78	72.42	−0.91	−0.15	−0.11
1861.5......	163.40	72.86	−0.13	+0.13	+0.20
1880.5......	163.98	73.10	+0.61	+0.18	+0.37
1901.3......	164.10	73.14	+0.76	+0.02	−0.10
1909.1......	164.28	72.94	+0.99	−0.26	−0.20
Yearly	motion	+0.010	+0.030		

	AB*				Differences from Uniform Motion	
	p	s	x	y	Δx	Δy
1835.2......	257°5	4″78	−0″33	+4″77	−0″10	−0″24
1846.0......	250.9	4.97	+0.22	+4.97	−0.03	−0.04
1861.5......	242.4	5.34	+1.03	+5.24	+0.11	+0.23
1880.5......	234.5	5.63	+1.84	+5.32	+0.09	+0.31
1901.3......	224.9	5.66	+2.71	+4.97	+0.06	−0.04
1909.1......	221.8	5.48	+2.89	+4.77	−0.11	−0.24
Yearly	motion		+0.044	0.000		

* Interpolated from Lewis' table, *Memoirs R.A.S.*, **56**, 214, 1906.

The motion of A, relative to both B and C—especially in y—shows a distinct curvature, which appears to be nearly equal in

[1] *Monthly Notices*, **67**, 140, 1906.

amount in the two cases, indicating once more that the center of gravity of the system is near the fainter component. The curvature is at present too small to afford a reliable determination of the mass-ratio; but within twenty years a good determination will be possible. For this reason careful measures of AC, especially of the position angle, should be made every year.

It might at first appear that our knowledge of the orbits of the spectroscopic binaries might help us here. But as only one component of each of these is bright, all that we can get from this is the value of $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$ (where m is the mass of the bright star, m_1 is t' at of its invisible attendant, and i is the inclination of the orbit). This may for convenience be called the "apparent mass." If $\frac{m}{m_1}=c$, the actual mass of the system is $(1+c)^3 \operatorname{cosec}^3 i$ times the "apparent mass." From Curtis' data we have for the stars in question:

	$a \sin i$	P	Apparent Mass
	km	d	
α_2 *Geminorum*	1,485,000	9.219	0.0015 ☉
α_1	1,279,000	2.928	0.0097 ☉

The "apparent mass" of the fainter component is six times that of the brighter; but the sum of the "apparent masses" of the two is only $\frac{1}{600}$ of the real sum of their masses, previously determined.

It is evident that, for one or both of the close pairs, c must be large or i small. Beyond this they are at present unknown, and hence the ratio of the "apparent masses" tells us nothing about the real mass-ratio.

If the planes of the secondary orbits are approximately coincident with that of the visual orbit (which seems plausible) and c is the same for the two close pairs, it must be about 6.5. If less than this for one pair, it must be more for the other.

This is a much greater preponderance of the primary than has appeared in any of the systems susceptible of reasonably exact investigation. The greatest well-established value of c is 3.0

in the case of Procyon.[1] To get as small a value in the present case it would be necessary to assume that the orbits of the close pairs are inclined less than 30° to the plane of projection—that is, that both are inclined at least 35° to the plane of the visual orbit, and that the ascending nodes of both on this plane lie near its descending node on the plane of projection. This seems rather improbable, and the difficulty increases if we assume smaller values of c. It is therefore probable that the "dark" companions in these two spectroscopic binaries are less massive, in comparison with their primaries, than is the case in any systems previously investigated.

However this may be, it is certain that the small "apparent masses" of these pairs are no valid indication of actual smallness of mass. This bears on the interpretation of numerous similar cases. The great range presented by the "apparent masses" of spectroscopic binaries has recently been discussed by Campbell[2] and by Schlesinger.[3] It is easy to show that the number of small "apparent masses" is much too great to be explained by the chance occurrence of small values of i. Schlesinger accounts for them by assuming that the masses of some of these stars are actually insignificant in comparison with that of the sun, and Campbell by assuming that in these instances the companions are of considerably smaller mass than the visible primaries.

The present case, which is so far the only one in which the actual masses of such pairs can be found, confirms the latter explanation and is inconsistent with the former.

Certain similar cases (e.g., *13 Ceti*, *κ Pegasi*) where one component of a visual binary is spectroscopically double, will further test this matter in future.

Meanwhile the clear evidence in this typical case makes it seem reasonable to regard the very small "apparent masses" as evidence of small *relative* mass of the unseen companion, rather than of minute *absolute* mass of the whole system.

PRINCETON UNIVERSITY OBSERVATORY
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[1] Boss, *Preliminary General Catalogue*, p. 267, 1910.

[2] *Lick Observatory Bulletin*, No. 181, **6**, 40, 1910.

[3] *Pub. Allegheny Observatory*, **1**, 147, 1910.

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. I

By FRANK SCHLESINGER

Almost all the determinations of stellar distances that have been published up to the present time have been made with instruments of short focal lengths, surprisingly short when we consider the accuracy that is demanded and has occasionally been attained in this class of work. Thus for example, the Yale heliometer, used by Elkin, Chase, and Smith to such good purpose, is only 2.5 meters long.

In 1902 some correspondence upon this subject passed between Professor Hale (then director of the Yerkes Observatory) and the present writer. It seemed very likely that, with the great focal lengths of modern refractors and the consequent increase of scale, it ought to be possible to bring about a considerable decrease in the accidental errors of observation, or else to reduce greatly the labor necessary to obtain results within a predetermined limit of error. The founding of the Carnegie Institution late in that year afforded an opportunity for putting this idea into practice, for among its first grants was one for this specific purpose to Professor Hale, who invited the writer to take charge of the experiment with the great Yerkes refractor. My connection with the Yerkes Observatory extended from May 1903 until I assumed new duties at the Allegheny Observatory in March 1905. In this interval Professor Hale had been succeeded in the directorship of the observatory by Professor Frost, who was equally interested in the outcome of this work, and who made it possible for me to complete it in its present form.

It is a pleasure to acknowledge here, at the outset of these papers, my obligations to those who have contributed so largely to them in one way or another: first, to the Carnegie Institution, under whose auspices the work was undertaken, the Institution assuming the principal items of expense, such as the salaries of

myself and of a computer, and the cost of a measuring engine; to the Yerkes Observatory and its successive directors, for putting at my disposal the facilities of the observatory, including one-quarter of the total nights with the 40-inch refractor, and for defraying all minor expenses, such as the cost of plates and recording blanks; to Miss Louise Ware, now of the Solar Observatory at Pasadena, who proved a most efficient assistant, and to whose conscientiousness the measures and reductions owe much of their merit; to Mr. Frank Sullivan, night assistant at the telescope, who aided in securing almost all the plates; to Mr. Philip Fox and Mr. Frank C. Jordan, who, at Director Frost's request and with Mr. Sullivan's assistance, secured the additional plates necessary to complete the series for certain of the stars; and finally to Mr. Robert H. Baker, who did some computing early in 1907, under a special grant to the writer by the Carnegie Institution.

AT THE TELESCOPE

That excellent photographs may be secured with a large visual refractor was first demonstrated by Mr. G. W. Ritchey,[1] who employed the method of placing a yellow color-screen or filter immediately in front of a plate that is sensitive to the yellow as well as the blue rays. I had expected to make use of this ingenious device for the parallax plates, but upon securing some photographs upon yellow-sensitive plates without a screen, and after comparing them with some of Mr. Ritchey's, I came to the somewhat surprising conclusion that isolated images upon the former are not inferior either in smallness or sharpness to those upon the screened plates.[2]

The reasons for this I have given in some detail in the *Astrophysical Journal*, **20**, 123, 1904. They are in brief that the plates employed, Cramer Instantaneous Isochromatic, act as their own filters for the region λ 4800 to λ 5200, as they are only slightly sensitive to this part of the spectrum. They have a strong maximum at λ 4500, but light of this wave-length is so much out of

[1] *Astrophysical Journal*, **12**, 352, 1900.

[2] Under the most favorable circumstances either kind of plate, taken with this telescope, will separate the components of a double star that are $0''.75$ apart.

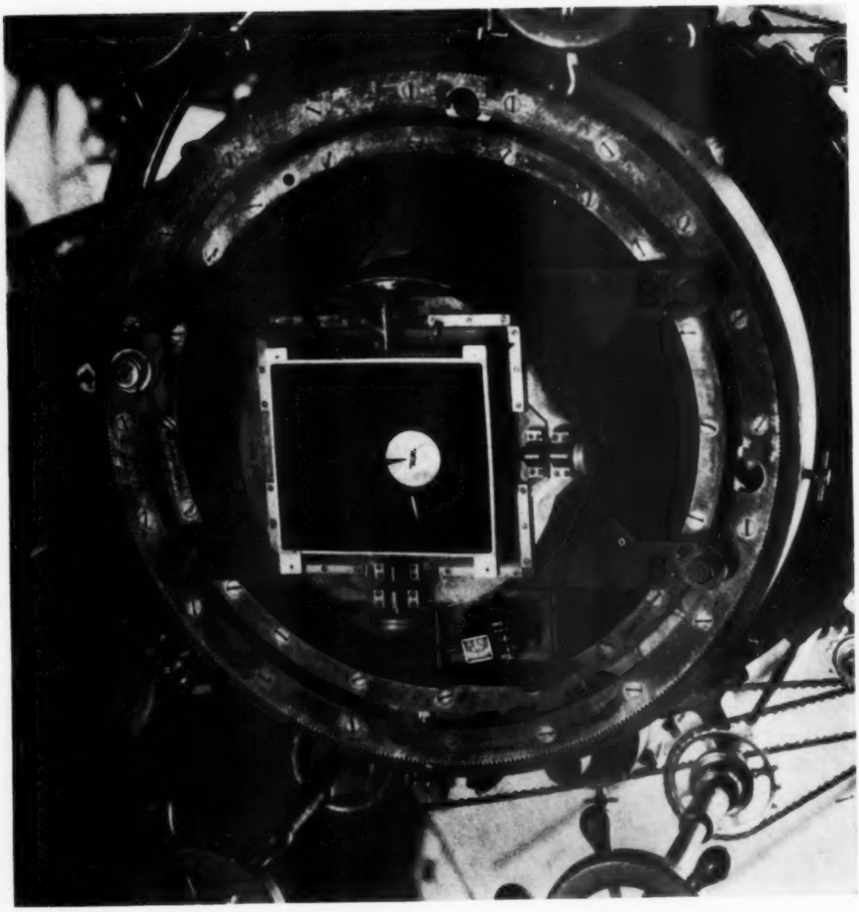
focus that it is spread into a circle about 2 mm in diameter by the time it reaches the plate, and is consequently much enfeebled if the source of light is a point. For exposures of moderate length the only part of the spectrum that is effective in forming the image is included between λ 5200 and λ 5700, a region that is all in fairly good focus for a proper setting of the plate. If the exposure be prolonged, the brighter stars upon the plate will begin to show halos several millimeters in diameter, due to the blue and violet light. Such an image is not suitable for accurate measurement, even though its nucleus remains well defined. This constitutes a disadvantage in the unscreened plates, since it is not possible, or at least not advisable, to include in the measurements as large a range of magnitudes as upon the screened plates. For parallax work, however, this is of no great consequence, because other considerations make it desirable that the comparison stars should not differ too greatly from each other in brightness, a point that will be referred to later.

The use of screened plates raises the question as to the effect of the screen's presence upon precise measures, and the distortions introduced by a plate of glass in front of the sensitive film. It was deemed better to avoid this difficulty rather than to devise methods for overcoming it, and for this reason all the plates here discussed were taken without a screen.

The above remarks refer to the filters that were then available. I had pointed out[1] in 1904 that "the best screen for stellar work would not be one which cuts out only the blue rays, but rather one which prevented the region λ 5200 to λ 5400 from reaching the plate." Since then Mr. R. J. Wallace, who had made the original screens without special reference to the Yerkes refractor, has had an opportunity for studying its color-curve in this connection, and has succeeded in producing screens that leave little to be desired. Not only do they intercept almost completely all that portion of the spectrum that is out of focus, but they introduce practically no absorption for the yellow and orange rays. Stellar images secured with these later screens are superior to the earlier in sharpness, and are more suitable for precise measurement. The

[1] *Astrophysical Journal*, **20**, 125, 1904.

PLATE XIX



Double-Slide Plate-Carrier for 8×10-Inch Plates as Attached to Yerkes Refractor

question whether screened plates or unscreened should be used for astrometrical purposes, in connection with a visual refractor, is thus reopened, particularly as I have recently shown that the errors introduced by photographing or measuring through glass are very small, even if ordinary glass has been employed, and are probably altogether negligible if the screen is made of worked glass.[1]

The first few plates obtained for the parallax work were secured with a double-slide plate-carrier for 4×5 inch plates. This had the advantage of being readily attached to the telescope, but it was soon found that the field subtended was too small for practical purposes and the more cumbersome 8×10 inch (20×25 cm) carrier was used instead. A photograph of this instrument appears herewith (Plate XIX), and a brief description of it is as follows: A small diagonal prism *P* is placed just in front of the photographic plate, and near its longer edge. The bundle of rays from a star that would otherwise form an image upon the plate are thus brought to the focus of a positive eyepiece *E*, in which are two fine spider-threads, respectively parallel and at right angles to the diurnal motion. The 40-inch telescope having been pointed by means of the slow motions until the threads exactly bisect the image, the observer keeps the star in this position, and compensates for irregularities in the driving clock and in refraction, by moving the eyepiece with the two screws *D* and *R*. These screws are at right angles to each other, the first moving the eyepiece and upper slide upon a second or lower slide that is controlled with the screw *R*. The upper slide not only supports the guiding eyepiece but the plate-holder as well. Consequently the plate follows the apparent fluctuations of the stellar images with the same faithfulness that the guiding star is held at the intersection of the spider-threads in the eyepiece, and all the stellar images upon it remain small and round. In order to afford greater facility in finding a guiding star after any desired object has been placed in the center of the field, the

[1] *Publications of the Allegheny Observatory*, **1**, 101, 1909. As early as 1903, Ludendorff (*Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, **15**, No. 49) had occasion to measure some plates through glass and concluded that the errors thus introduced, while appreciable, are not so large as to prohibit measurements made in this way.

guiding eyepiece is provided with a motion in the direction of the long edge of the plate; and with the same object in view the lower slide is mounted upon a circular casting that may be rotated in position-angle. This circular casting is provided with a coarse circle, graduated at every fifth degree; the index itself is graduated to single degrees. It is thus possible to set the instrument within a fraction of a degree, and this is ample for the present purposes. The whole plate-carrier, which weighs about 43 kilos (96 lbs.), is attached to the tail-piece of the telescope by means of four stout bolts *B*, *B*, and its position is fixed by two dowel pins. The tail-piece itself may be racked toward or from the 40-inch objective, and it is in this way that the film of a plate is adjusted to the focal plane. The plate-holder is of brass, and loads from the back, as every plate-holder should that is to be used in precise work, for then the plate is supported along its entire perimeter by a suitable ledge, against which it is pressed by several weak springs.[1]

The optical center of a photograph may be defined as the foot of a perpendicular let fall from the center of the objective on the plane of the plate. It is necessary (with photographs intended for accurate measurement) that we should know where the optical center lies, and that it should not be too far from the geometrical center of the plate. For this purpose an aluminum plate was made of the same size as the photographic plate, and it was fastened in the holder in precisely the same position that the photograph itself ordinarily occupies. To this aluminum plate a small telescope was fixed, approximately perpendicular to its plane and projecting through a hole in its center. In its focus were two parallel wires, which could be adjusted so as to be tangent to the image of the 40-inch objective. The plate was now removed from the holder and turned 180° around the optical axis of the large telescope. If the little telescope had been accurately perpendicular to the plane of the plate, and if the latter had been perpendicular to the

[1] The plate-carrier was designed and constructed by Mr. Ritchey, and is the one used by him to obtain his lunar photographs; a few additions for which the writer is responsible were made to meet the requirements of the parallax work, consisting of the graduated scale on the guiding eyepiece slide, the graduated circle, the dowel pins to fix the position of the whole carrier, the metal plate-holder, and a device, to be described presently, for partially occulting the light of a bright star.

line joining its center with that of the 40-inch objective, the image of the objective would have again been tangent to the two parallel wires in the focus of the little telescope. The amount by which this fails to be the case can be estimated with sufficient accuracy and gives the position of the optical center. This method furnishes an accurate and convenient means for adjusting the "tilt" of the plate, and the writer has used it to advantage with several telescopes.

In the present instance it was found that the optical center lay about 6 cm (11 minutes of arc) east of the geometrical center, and 5 cm (9 minutes of arc) south of it, the telescope being west of the pier and directed to the intersection of the meridian with the equator. These quantities are somewhat larger than we should expect. No adjustment for tilt had been provided in the design of the plate-carrier, reliance having been placed on the mechanical precision of the tail-piece, etc. A search was made to locate the cause of the divergence and it was found to lie in the inequality of the four tubes by which the tail-piece is racked toward and from the objective. The upper right-hand one of these was found to be nearly 2 mm longer, and the opposite one the same amount shorter, than the two others.[1]

This mechanical defect might have been corrected by removing the tubes and making them the same length; but this would have occasioned considerable inconvenience and expense, and would have necessitated a readjustment of the solar and stellar spectrographs that are in constant use with this telescope. A simpler plan would have been to alter the plate-carrier so as to give it an equal tilt in the opposite direction. However, neither of these changes was carried out, the small field subtended by the photographs making it unnecessary in the present work. With a set of comparison stars as they would ordinarily be distributed upon the plate, the deduced position of the central star would be affected by less than $0\rlap{.}''01$, if the optical center should change its position by as much as 3 cm. Therefore all that is necessary is that the tilt should not change beyond this limit. The test above described was carried out twice with an interval of two years, and was

[1] The centers of the tubes form a square that is 64 cm on each side.

found to give substantially the same results. It was furthermore found that the tilt remained the same whether the tubes had been racked in or out. The flexure of the massive tube of the telescope may also introduce a slight change in the tilt, but this need be given no further consideration in the present work, since all the plates were taken with the telescope west of the pier and not far from the meridian; and the flexure is therefore always the same for any one star.

As the time just after dark is very valuable in parallax work, especially if the regions are to be photographed near the meridian, it was desirable to devise some rapid method for setting the plate in focus, without suffering the delay incident to securing the usual "focus plate." For this purpose a rigid aluminum plate was fixed in the position ordinarily occupied by the metallic plate-holder. In its center an adapter was mounted, into which fitted a 750-power eyepiece, whose tube was graduated into millimeters. A focus plate having first been secured and developed, the tail-piece was racked in or out so as to place the film in the focal plane. The aluminum plate was now attached and the eyepiece pushed in or out until a star appeared in good focus. The reading of the scale upon the eyepiece was then recorded. These operations were repeated on several evenings and the mean of all the eyepiece readings was adopted as standard. In order to set the plate to focus on a subsequent evening, without securing a focus plate, all that it is necessary to do is to focus the eyepiece upon a star, to obtain the reading on its scale, and then to rack the tail-piece in or out by an amount equal to the difference between this reading and the standard. This not only saves much time but it is actually more accurate than securing a focus plate; for it was found that the standard readings for the eyepiece scale, as deduced from different focus plates, varied by as much as 2 mm; while the eyepiece could be set to focus visually with considerably greater accuracy, if the definition were not too poor.

Later in these papers we will discuss the amount of error that is incurred by a maladjustment of the plate to focus.

Perhaps the most prolific source of error in early parallax determinations, particularly those made by means of photography,

has been what Kapteyn calls the "hour-angle error." The complete explanation of its origin has not yet been given, but it is doubtless due, partly at least, to the cause assigned by Rambaut, who first called attention to this error under the name of "Atmospheric Dispersion."[1] Let us suppose that we have two stars in the same field, one of which is white and the other red. If the distance between the two stars is measured at such an hour-angle that the red star appears *above* the other, we shall get this distance too small, because the coefficient of refraction for blue rays, which are relatively richer in the white star, is greater than for the red rays. On the other hand, if we measure the distance between them at an hour-angle for which the red star appears *below* the other, we shall get a measured distance that is too large. In the present work, this source of error is less to be feared than is usually the case, since only a narrow region of the spectrum (λ 5200 to λ 5700) is effective in producing the images, no matter what the color of the star may be. It would be entirely feasible to eliminate this error, as well as any others that may depend upon the hour-angle, by investigating its effect with a properly planned series of plates, for each region under observation. But it is perhaps well to avoid the error altogether rather than to attempt to evaluate it; and this may be done by photographing the region, as Kapteyn has repeatedly urged, always in the same hour-angle and preferably near the meridian. This has accordingly been done for all the stars here discussed. It was of course not practicable to comply strictly with this condition, for the time with the Yerkes refractor is very precious; so that I did not wait until the next region came to just a certain hour-angle, but secured as many plates as I could during the night, even though some were taken, in extreme cases, as much as two hours from the mean hour-angle. An effort was made, however, to keep the mean hour-angle for the morning plates equal to the mean hour-angle for the evening plates of any one region. It can easily be shown that this procedure would almost entirely eliminate this error, if it is proportional to the hour-angle itself; and this must be approximately the case if the plates are taken not too far from the meridian. But even this

[1] *Monthly Notices*, **55**, 123, 1895.

less rigid condition could not always be complied with, and for a few of the regions the mean of the morning hour-angles differs considerably from that for the evening plates. In order that the reader may see for himself how these hour-angles run, this datum is set down for each plate in the tables of observations to be given later.

Another possible source of error is "optical distortion." This may be defined as an error in the place of stellar images, depending upon the orientation of the objective with reference to the telescope tube. If the objective were perfect in shape and homogeneity, of course no such error could exist. This subject has been studied by several astronomers, and with especial care in the case of the two astrographic refractors at Helsingfors and the Cape of Good Hope. In both cases it was concluded by Dr. Furness[1] and Professor Jacoby[2] that no optical distortion of appreciable size exists. But it would be unsafe to extend this conclusion to other objectives, and this remark acquires particular force for the Yerkes refractor. The great weight and diameter of the lenses, and the consequent sag when the telescope is pointed near the zenith, produce their effect upon the photographic images,[3] rendering them very slightly triangular, the vertices corresponding in position-angle to the three supports upon which the objective rests. This triangularity doubtless causes the bisections to be made at a slightly different place than if the image were perfectly round. The error varies with the magnitude of the star, since the triangularity is more pronounced with intense images and is in fact not at all apparent when the image is faint. The effect on the measures is in any case small, but it is very essential in parallax work to take it into the reckoning, where quantities of $0\overset{''}{.}01$ become important if they are systematic. For this reason, it was decided not to reverse the telescope with reference to the mounting, but to observe with the tube always on the west side of the pier. For reversing a telescope of this form is equivalent to rotating the objective and its cell 180° around the optical axis,

[1] *Publications of the Vassar College Observatory*, No. 1, p. 73, 1900.

[2] *Contributions from the Observatory of Columbia University*, No. 19, p. 74, 1902.

[3] A trace of the effect of sag seems also to be present in visual observations. See the paper by Keeler in the *Astrophysical Journal*, **3**, 154, 1896.

as referred to any configuration in the sky. If this were done, it is obvious that the distortion would at some times shift the images apparently to the east, say, and at others to the west. As the tendency would be to take all the evening plates with the tube to the east of the pier and all the morning plates with the tube to the west, almost the whole effect of the distortion would enter into the parallax determinations. It has been suggested by Jacoby that the objective be rotated 180°, around the axis of the tube, each time the telescope is reversed. This would eliminate the error, but as no such rotation was provided for in the construction of the telescope, and as it is questionable whether it would be advisable to rotate this objective, the only alternative was to avoid reversing the telescope.

There are other reasons why I decided to observe with the tube always west of the pier: first, the driving clock was found to perform somewhat better in this position than in the other.[1] Observing from the west side, rather than the east, is in any case preferable, as it interferes with the work in the morning hours less than in the evening, and this is as it should be. Again, restricting operations to one side of the pier and to the neighborhood of the meridian preserves the constancy of conditions (so desirable on general principles in parallax work) as far as it is possible to do so.

Toward the beginning of the work the rule of observing only on one side of the pier was violated for a few plates. Attention will be called to each of these cases in the tables of observations.

As the focal length of the telescope is 19.36 meters, one millimeter on the plate corresponds to 10″6. The dimensions of the plates employed (20 cm by 25 cm) therefore correspond to about 35 and 45 minutes of arc. In order to secure suitably situated comparison stars in sufficient number within so small an area, it is usually necessary to use stars down to about the tenth magnitude, and for this purpose exposures of 5 minutes are ordinarily sufficient. With so long a focal length the size of the images varies greatly with the atmospheric conditions, and it is therefore

[1] The remarkable faithfulness of this delicate mechanism, used in connection with so massive an instrument and in temperatures ranging from +30° to −30° C., is a source of constant admiration to all who have had the privilege of working with this telescope.

possible, under the best conditions and when the field is a rich one, to reduce the exposures to 2 minutes. On the other hand, when the transparency is poor, and more especially when the definition is bad, it is necessary to double the normal exposures.

The duration of exposure is fixed by the brightness of the comparison stars, rather than by that of the parallax star.[1] The latter is usually considerable brighter than the former, occasionally by as much as seven magnitudes for some of the stars here investigated. But even when the difference is only two magnitudes it becomes a matter of difficulty, on a plate for which the comparison stars are sufficiently strong, to measure the overexposed and broad image of the parallax star. Some device for enfeebling the light of a bright star is therefore a necessity. To this end, I suggested several years ago[2] that the film might be rendered less sensitive in the small area upon which the bright image will fall, by local washing with some colored fluid. A somewhat similar but more simple device was suggested to me with some hesitation by Dr. H. M. Reese: to expose the plate in the usual way, to develop first until the image of the bright star just appears, to "fix" the plate at this point only, by the use of a drop or two of hyposulphite, and then after washing to continue the development so as to bring out the images of comparison stars. Dr. H. N. Russell[3] has recently put in front of the photographic plate a sheet of plane-parallel glass, in the center of which, and nearly in contact with the film, is mounted a yellow patch of absorbing material. This appears to have worked well except as to permanency, a matter that there should be no great difficulty in overcoming.

None of these devices is quite satisfactory for the purpose here in view, for it is not only necessary to reduce the image of the parallax star to such an extent as to make it measurable, but it is hardly less desirable to be able to control its brightness within narrow limits, say two or three tenths of a magnitude, in order that its image on the developed plate may be closely equal in inten-

[1] For the sake of brevity, we shall use this designation for the star in each field whose parallax we desire to determine.

[2] *Astrophysical Journal*, **10**, 243, 1899.

[3] *Ibid.*, **26**, 147, 1910.

sity to the mean for the comparison stars. The reason for this lies in the "guiding-error," which is perhaps the largest item of accidental error still outstanding in general photographic work. This is due to the impossibility of keeping the images of the stars exactly stationary upon the plate during the exposure. They will all wander more or less from their mean positions, and while these excursions are equal in amount and direction for all the stars upon the plate, they are registered to a different degree for bright and for faint stars. Consequently the position of any image will depend to some extent upon its intensity. Now the parallax computations may roughly be said to consist in subtracting for each plate the mean of the measures upon the comparison stars from that upon the parallax star. It is evident, therefore, that the effect of guiding-error will be almost entirely eliminated if the magnitude of the parallax star can be reduced to the mean of the magnitudes for the comparison stars.

With this in mind I decided to try some mechanical method for occulting the light of the parallax star, and mounted an ordinary photographic shutter, about 4 cm in diameter, in the center of the plate-carrier. This was operated in the usual way by a tube and a hand bulb. The exposure began with the shutter closed, so that the light of the parallax star did not reach the plate. The observer then opened the shutter for a few seconds at a time and at intervals symmetrically distributed over the whole exposure. The developed plate showed excellent images for the parallax star, in no way distinguishable in appearance from those of the comparison stars. I had feared that the effect of diffraction due to the shutter might be visible, but no trace of this appeared. When, however, we measured the four plates (each with three exposures) secured in this way, we found that the probable error of the relative position of the parallax star was threefold that for ordinary plates; that is, plates on which the parallax star was, to begin with, about as faint as the comparison stars. The explanation of this is the strong tendency of the observer to avoid opening the shutter to the parallax star unless the guiding star is exactly bisected by the cross-wires; but the guiding star usually shows a tendency to drift off the wires in a definite direction, let us say to the right, for any

one plate. Consequently the position of the parallax star, as obtained by means of the shutter, corresponds closely to the intersection of the cross-wires, while the comparison stars correspond to points a little to the right.

This experiment indicated the necessity for occulting the star automatically and continuously without the intervention of the observer. Accordingly a rotating disk with an opening of variable aperture was constructed, something after the fashion of an Abney disk photometer. This is mounted directly in front of the photographic plate, with its center a little below that of the plate itself, as shown at *O* in Plate XIX. The disk is rotated perhaps six or eight times a second, by means of a small electric motor *M* just outside the plate-carrier, the motor being connected with the disk by means of a cord belt. The two halves of the disk can be rotated with respect to each other and then clamped, leaving a clear sector of any angular aperture desired up to nearly 180°. A scale and an index are provided so that the angle of opening can be set with accuracy. The whole device can be put into place or removed in a minute or two.

The method of using the rotating disk is as follows: the telescope is pointed so as to bring the parallax star to the center of the field, just above the axis of the disk. The opening of the latter is then set, let us say, at 36°, and the disk is put into motion. The exposure now begins; it is evident that only one-tenth of the light of the parallax star reaches the plate in several hundred intermittent exposures, lasting a few hundredths of a second each. Since the jaws of the opening are radial, it is obvious that this fraction of one-tenth is independent of the relative positions of the star and the disk, and of the speed at which the latter is rotated. The parallax star will, in this case, be apparently reduced in brightness by 2.5 magnitudes. For each region observed the amount of the opening was determined once for all by experiment, so as to reduce the apparent magnitude of the parallax star to the mean of the comparison stars.

This method was found to be practicable up to reductions of 7 magnitudes, corresponding to an angular opening of a little more than 0°.5. For smaller openings than this, that is, for greater

reductions, the uncertainty in setting becomes a considerable fraction of the whole opening, since the graduated scale is necessarily small on a disk that must not cover too large a portion of the photographic plate. The images thus obtained, like those with the photographic shutter, show no trace of diffraction effects due to the disk, and their measurement proves to be as accurate as upon unobstructed images of fainter stars. Some of the parallax stars in the present program are not very much brighter than the comparison stars, and for these the disk was not used. For similar work in the future, I should use the disk in relatively more cases than I have here, extending it to those fields in which the parallax star is only slightly brighter than the comparison stars. For experience has shown that the uncertainties in the measured positions, doubtless chiefly because of guiding-error, are considerably greater for a field in which the parallax star is as little as one magnitude brighter than the comparison stars, than for a field in which the parallax star was reduced by the rotating disk to approximate equality with them.

The question as to how many exposures should be made upon each plate is an important one. At the outset I decided provisionally upon three; and later, when opportunity was afforded to investigate this matter from a discussion of measures made upon the plates, this number was definitely adopted as being about the most economical. In common with other observers, I found that measures of exposures upon the same plate have a tendency toward better agreement than between exposures on plates secured on different nights. From this point of view alone, therefore, only one exposure should have been impressed upon the plate. But the time consumed in setting the telescope upon a region, finding the guiding star, etc., is large compared with the length of the exposure itself, and consequently three exposures can be secured in less than double the time that a single exposure would require. It consumed an average of about fifteen minutes to change plates in the holder, to point the telescope to a new field, to set the scales on the finding eyepiece slide, the position circle, and the rotating disk, and to find the guiding star. Thus for five-minute exposures the first was obtained after a lapse of about twenty minutes, and

the third after a lapse of about thirty-two minutes. For longer exposures the gain in time is of course relatively less.

The complete operation at the telescope was, therefore, as follows: Shortly before dark the focus was obtained by the visual method described. The telescope was now pointed to a star in the program that was then on or near the meridian. The guiding for the first exposure was executed by myself; the guiding eyepiece slide was then moved to the right or left so as to secure a second exposure upon the plate separated from the first by 5 mm, and the guiding for this was done by Mr. Sullivan. At its completion the slide was moved 5 mm to a third position and the guiding was resumed by myself. While Mr. Sullivan was guiding for the second exposure, I made the necessary notes and selected the next region to be photographed. Only one plate-holder was available, and at that time there were no facilities for changing the plates in the dome; so that it was necessary to take the holder to a dark-room in the observatory below, to store the exposed plate, and to put a fresh one in the holder. While I was doing this Mr. Sullivan pointed the telescope to the next star, from the information given on the "observing-card,"[1] rotated the dome, and put the rising floor into proper position. This process was repeated until about ten o'clock, after which time, and until well into the morning, it is not profitable to continue this work. The "parallax factors" of stars that are on the meridian near midnight are small (except in rare cases) and plates taken under these conditions would add little to the observational material. Accordingly the opportunity was taken to develop a few of the plates just secured, in order to make certain that all was going well. The telescope was then used for other photographic work (such, for example, as securing plates of "loose" clusters) until about two o'clock, when the parallax work was resumed. It will be seen that Mr. Sullivan's assistance greatly facilitated these operations; it increased the output of

[1] This card gave the position of the parallax star, the settings for the position circle, the guiding eyepiece slide, and the opening in the rotating disk, the normal length of exposure, and other information, such as a diagram of the stars in the field, that would enable the observer to begin the exposure with as little loss of time as possible.

the telescope, as compared with what one alone would have been able to accomplish in the same time, by at least 30 per cent.

The plates that have been measured and reduced for the purposes of the present papers number 327, and they relate to 25 different regions. Of these plates 201 were secured by myself, and the remaining 126 by Messrs. Jordan, Fox, and Sullivan. In addition, many other parallax plates were obtained successively by myself, by the observers named, and, since September 1909, by Dr. Frederick Slocum. These are to form the basis of subsequent studies of stellar parallax at the Yerkes Observatory.

Allegheny Observatory
December 1910

THE SOLAR ROTATION FROM THE MOTION OF THE FACULAE ON THE DISK (1906–1908)

By STANISLAS CHEVALIER

The history of this subject is short and well known. Only two investigations have to be considered, namely, that of Wilsing of the Astrophysical Observatory of Potsdam, who inferred that the rotation was uniform for all latitudes; and that of Stratonoff who, contrariwise, found that the phenomenon known as the equatorial acceleration extended to the stratum of the faculae. Although the conclusions of the latter were actually beyond doubt, it was clear that a new examination of the problem which should check, and increase the precision, if possible, of the values thus obtained, would not fail to be useful. Moreover, the recent investigations of Messrs. Hale and Adams upon the rotation of the flocculi and that of the reversing layer call for a more accurate measurement of the rotation from the faculae. It is for this reason, on the advice of Mr. Hale, that we have attempted to derive from the photographs of the Observatory of Zô-sè a new determination of that rotation.

Preparation of the photographs.—We began by examining all the negatives of the years 1906–1908, in order to select those which would be able to furnish data for this work. It is necessary to find two photographs separated by an interval of about twenty-four hours, both of which are as sharp as possible, to permit of recognizing the same faculae with certainty on both of the plates. It does not suffice to find on them one and the same group, but it is necessary to be able to determine the same small facula and the same detail distinctly marked in the group. For this work of reconnaissance it is necessary to examine the two photographs side by side very carefully, with the assistance of eyepieces having greater or less power. These identical details once found must be marked in red ink on the varnished film of celluloid with which we cover all our negatives of the sun. This point should be marked as accurately as possible and in the same manner upon the two photographs, for the measurer has to determine the heliographic position of the

point. The method of pointing is certainly not ideal, and it introduces a new source of error, which will be of the same order as errors of measurement. But with objects so delicate and difficult of recognition, it was absolutely indispensable.

This work of identification is important and difficult, for the observer has no guide except the similarity of the form, and occasionally the relative position of the spot. The mobility of the faculae is furthermore such that it is generally impossible to be absolutely certain of the identity of the two faculae. It is necessary to content oneself with a greater or less degree of probability. It is, therefore, inevitable that one must make some errors of identification. Sometimes the faulty identification of the faculae marked appears on the first attempt at measurement: then there is nothing to do but to discard it without giving it further attention. But there are cases where the error remains doubtful, where one does not know whether he is dealing with a facula having a very large proper motion, or whether there are two faculae of the same group more or less closely resembling each other. The observer is in the well-known situation where the extreme residuals must be rejected.

Measurement of the plates.—The sources of error, and very appreciable ones, are numerous and inevitable in the measurement of the rotation of the sun from faculae. The proper motion of the faculae, their real changes of form, which occur often and rapidly, their changes of form due to perspective, the fact that their position is always so near the limb, which interferes with the precise determination of their longitude—these are all necessary causes, to which we are obliged to add our errors of pointing. One therefore cannot hope to obtain anything more than a tolerable accuracy in the measures, except by making a large number of observations in each zone.

It accordingly becomes indispensable to adopt some method of measurement which permits the determination of many thousands of heliographic positions without excessive labor. Since the beginning of our work on the sun we have employed a very simple procedure in measuring heliographic positions on our photographs. It consists in placing upon the image of the sun a transparent

chart of the solar meridians and parallels, such as would be imprinted upon the image if they really existed. With the chart selected, and accurately fitted upon the solar image, the heliographic position of any object whatever upon the surface is obtained by a single reading. As our first charts seemed to be too imperfect, a series of new ones on a larger scale (disk of 40 cm diameter) were prepared for the work. The series includes fourteen charts, each differing from the preceding one by increasing the latitude of the center by half a degree. From these charts the parallels are traced by alternate degrees from zero to 42°, and the meridians are traced at the same interval up to 80°. At the center of the chart one degree is equivalent to 3.5 mm, and the error of the ruling does not exceed 0.1 mm, or 0°03. Near the edges the relative error is naturally somewhat larger. These charts, drawn by our draftsman, Mr. F. Tsang, have appeared to be very satisfactory.[1] Every one of them has been photographed several times, to conform to the size of the solar image at different dates.

The maximum difference which is permitted to occur between the latitude of the center of the solar disk and that of the center of the chart employed is 15′. Its effect, which is nearly zero on the longitude, is more sensible on the latitude, but it is possible to cancel it by a double measurement with two charts between which the true latitude of the center is comprised. Mr. Hale, to whose examination I submitted one of the charts, as well as the plan of employing them in our work, replied as follows:

> I requested Professor Seares, superintendent of our computing division, to make a critical examination of the method of measurement and to report the results to me. . . . . Professor Seares believes that if a sufficient number of measuring scales is available, the error of measurement of a position should not exceed 0°3 to 0°5. It seems probable to me that results obtained by this method will be sufficiently precise, especially in view of the nature of the faculae and their rapid change in form.

The sole difficulty of the measurement consists in accurately fitting the chart to the solar image so that the equator of the chart coincides with that of the sun, and the central meridians and the

[1] An example of these charts and a description of the method used in drawing them may be found in Vol. IV of the *Annales* of the Observatory of Zô-sè.

edges are accurately superposed. The two photographs are then placed between the clamps of a support which keeps them pressed one upon the other, before the eye of the observer. Using a magnifier, he has only to verify the coincidence and read the heliographic position. To diminish errors, but particularly to avoid the introduction of an error of 2° in the reading, which might easily be made, all the measures are done in duplicate and are verified in case of doubt. All these measures were made by Mr. Sinow Zeng, chief of the bureau of solar measures, and upon examination of the double series of independent measures, it is manifest that the readings were very well made.

Calculation and formation of the tables of observation.—Five hundred and seventy-two plates combined in pairs have been utilized in the work, from which 5216 heliographic positions of faculae have been derived. The positions of the same facula picked out on two plates separated by an interval of time θ, generally about 24 hours, give by a simple difference the observed motion of the facula during the time θ. The observed motion in longitude multiplied by the ratio $24:\theta$ gives the synodic rotation of the sun in 24 hours. To pass from this to sidereal rotation, it is only necessary to add the projection on the solar equator of the motion of the earth during the twenty-four hours. The 5216 observations of the faculae have furnished 2608 measures of the diurnal sidereal rotation of the sun. All the observations have been reduced to tables of nine columns, containing: the date of the photographs and the factor $24:\theta$; the successive number of the observed faculae, arranged by dates of the plates and upon each plate according to the latitude; the mean latitude and the observed variation in latitude; the distances to the central meridian measured on the two photographs, and the motion in longitude, which is deduced from it; the synodic rotation in 24 hours, and, finally, the diurnal sidereal rotation, which we shall designate by ξ, as is customary. The examination of the tables has led us to eliminate 33 values, for which the variation in latitude, combined with the variation in longitude, implied a proper motion of the facula equal or superior to 3°. The reason for adopting that limit was that a summary examination gave $0^\circ.5$ as the mean error of a measure.

Procedure, and result of observations.—To make use of the 2576 measures of the rotation which remain, it was necessary to construct new tables which would be arranged, not chronologically, but according to latitude. For these abridged tables, which will be published in the *Annales* of the Observatory of Zô-sè, we have adopted the division into zones of 5°, from the equator to the two poles. Besides furnishing a number of zones, limited but sufficient, this division has the further advantage of being more readily adapted to a comparison between our results and those of other observers. We shall give here only the final results of the observations, i.e., the rotation ξ obtained for each zone of five degrees, the number of observations made in the zone, the mean latitude of the faculae measured, the mean error of one measure of rotation, and the probable error of the mean. For completeness, we shall add the mean of the variations of latitude observed in each zone.

Latitude Zones	Number of Observations	Mean Latitude	ξ	Probable Error of the Mean	Mean Error of an Observation	Mean Variation in Latitude
+30° to +25°.....	42	26°.6	13°.900	±0°.113	±0°.495	0°.50
25 to 20......	97	22.2	14.168	0.088	0.582	0.55
20 to 15......	246	17.6	14.209	0.049	0.521	0.50
15 to 10......	371	12.4	14.279	0.039	0.508	0.50
10 to 5......	312	7.9	14.451	0.044	0.530	0.45
5 to 0......	101	3.0	14.471	0.071	0.479	0.55
0 to − 5......	97	3.3	14.521	0.076	0.505	0.49
− 5 to −10......	249	7.9	14.418	0.050	0.531	0.51
−10 to −15......	375	−12.5	14.390	0.036	0.467	0.43
−15 to −20......	359	−17.4	14.251	0.040	0.508	0.44
−20 to −25......	222	−22.2	14.209	0.051	0.507	0.53
−25 to −30......	104	−26.8	14.042	0.063	0.432	0.54

In order to give a still clearer idea of the solar rotation from faculae in the above table, we have drawn a curve of the values in the column under ξ. The second curve of that figure, which is entirely regular, was drawn from the means of the same observations, but grouped by zones of 10°.

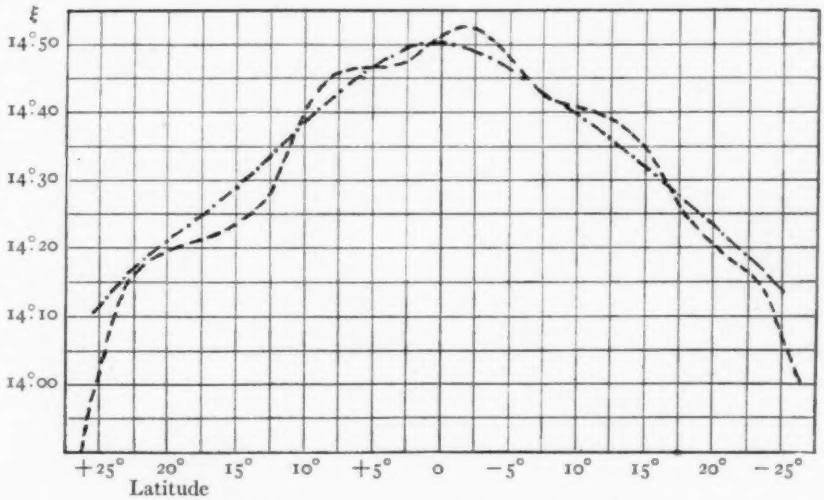
Briefly summarized, the conclusions which may be drawn from the curves are as follows:

1. The equatorial acceleration is so clearly indicated in the whole zone from +30° to −30°, over which our observations extended, that it is absolutely impossible to fail to recognize it.

2. The acceleration is very regular, even on the curve of five-degree zones, and its irregularities may properly be assigned to errors of observation.

3. The variations seem to be slightly less in the vicinity of the equator than in higher latitudes. This is furthermore entirely natural, since the curve passes through a maximum at about 0°.

4. The difference between the two hemispheres is so clearly indicated that it is difficult to attribute it to an accidental arrangement of errors of observation. All the values of ξ in the southern hemisphere are larger than the corresponding ones in the northern hemisphere. There is but a single exception for the zone $+5°$ to $+10°$, and further, the general course of the curve leads us to think that the value of the zone $+5°$ to $+10°$ is a little too large. It therefore seems highly probable that during the period of three years, 1906–1908, the mean rotation has been actually a little more rapid in the southern hemisphere than in the northern.



Solar Rotation from motion of the faculae (1906–1908)
— — — from 5° zones; —·—·— from 10° zones

5. There is another very remarkable difference between the two hemispheres. The variation of velocity as a function of the latitude is more rapid in the northern hemisphere than in the southern.

6. According to the curve based on the observations by five-degree zones, the maximum velocity would not be found exactly at the equator, but a little south of it.

The last point is, however, altogether doubtful; for it depends only upon the values obtained for the equatorial zone, which are less certain on account of the more limited number of observations.

In grouping the observed values of ξ by zones of 10°, we obtain in each hemisphere three points which are nearly on a right line. Their arrangement entirely contradicts the remark under 6, although it confirms the others.

Despite the fact that the differences between the two hemispheres seem to be real, as they are neither certain nor constant, they ought not to prevent us from combining the zones of the same latitude north and south to obtain a mean value which should be more accurate and more assured. This mean is given in the following table. We have included the mean latitude corresponding to the mean value of ξ, and the number of days necessary for a complete solar rotation with the velocity assigned.

Zones	ξ	Probable Error	Latitude	Period of Rotation in Days
0° to 5°........	14°495	±0°033	3°2	24°836
5 to 10........	14.436	0.021	7.9	24.937
10 to 15........	14.335	0.017	12.4	25.129
15 to 20........	14.236	0.020	17.3	25.288
20 to 25........	14.197	0.032	22.2	25.357
25 to 30........	14.001	0.041	26.8	25.712

Two formulae are almost equally well adapted to express this result, the one a function of the square of the sine, and the other a function of the cosine of the latitude, viz.:

$$\xi = 14^\circ.470 - 2^\circ.268 \sin^2\lambda = 868'.2 - 136'.1 \sin^2\lambda;$$
$$\xi = 10^\circ.140 + 4^\circ.331 \cos\lambda = 608'.4 + 259'.9 \cos\lambda.$$

These two formulae give sensibly the same velocity of rotation for the different latitudes, within the limits of observation, but it is clear that they could not be extended to a higher latitude.

Comparison of our results with those of other observers.—As there is only one other attempt at the measurement of solar rotation by

the faculae, that of Stratonoff of the Observatory of Pulkowa, we must first of all compare our results with his. Since Stratonoff's memoir is not available to me, I will copy his values from the volume by Messrs. Hale and Fox, *The Rotation Period of the Sun as Determined from the Motions of the Calcium Flocculi*, No. 93 of the *Publications of the Carnegie Institution of Washington*. The following table gives a comparison of the two series of determinations at Pulkowa and at Zô-sè. We have added the number of observations made in each zone to the value of the rotation found for the zone. To the table of mean values based on the same observations for the two hemispheres we have given a column for the probable error of the mean.

Zones	Northern Hemisphere				Southern Hemisphere			
	Pulkowa		Zô-sè		Pulkowa		Zô-sè	
	Number	ξ	ξ	Number	Number	ξ	ξ	Number
0° to 5°	9	14°.62	14°.471	131	...	.....	14°.521	97
5 to 10.....	39	14.61	14.451	312	9	14°.63	14.418	249
10 to 15.....	125	14.34	14.279	371	67	14.26	14.390	375
15 to 20.....	110	14.14	14.209	246	124	14.21	14.251	359
20 to 25.....	124	14.21	14.168	97	137	14.17	14.209	222
25 to 30.....	109	13.97	13.900	42	101	14.20	14.042	104
30 to 35.....	15	13.50	......	...	34	13.65	......	...
35 to 40.....	...	.....	......	...	24	13.61	......	...

Zones	Means of Both Hemispheres			
	Pulkowa		Zô-sè	
	ξ	ϵ	ξ	ϵ
0° to 5°	14°.62	±0°.127	14°.495	±0°.033
5 to 10.....	14.61	0.061	14.436	0.021
10 to 15.....	14.31	0.044	14.335	0.017
15 to 20.....	14.18	0.036	14.296	0.020
20 to 25.....	14.19	0.036	14.197	0.032
25 to 30.....	14.08	0.040	14.001	0.041
30 to 35.....	13.60	0.059	......	.....
35 to 40.....	13.61	0.086	......	.....

It seems evident, owing to the much larger number of observations, that our series marks an actual progress in the study of the

question. The superiority of our series follows less from the comparison of probable errors than from the regularity of the values obtained. If the accordance of the two series is not complete, it is nevertheless very satisfactory, at least for the zones where the Pulkowa series has a sufficient number of observations. The discordance is hardly worth considering, except in the two equatorial zones from 10° to 0°, but for those two zones Pulkowa has only a very small number of measures, forty-eight for one and nine for the other. The value 14°61 for zone 5° to 10° is in accordance neither with those of a higher latitude, nor with those of the zones 0° to 5°.

Comparison with the rotation derived from spots.—We now should compare the rotation we have determined from the faculae with that from the motion of spots. The spots lie in the photosphere, while the faculae float at a great distance above them. Do these two strata share in the same rotation? This comparison ought to settle the question. Unfortunately in spite of all the work on the spots and the solar rotation according to the motion of the spots, we have no expression of that rotation of which we may be truly certain. The formulae proposed, following that of Carrington, are numerous, but they are in very poor agreement among themselves, as well as with those which we have derived from the movement of the faculae. They are:

Carrington,	$\xi = 865' - 165' \sin \frac{7}{4} \lambda.$
Faye,	$\xi = 862' - 186' \sin^2 \lambda.$
Spoerer,	$\xi = 512'.9 + 347'.9 \cos \lambda.$
Tisserand,	$\xi = 857'.6 - 157' \sin^2 \lambda.$

The most extensive work on the subject, and that which has involved the most numerous observations, seems to be that of Mr. and Mrs. Maunder, entitled "Solar Rotation Period from Greenwich Sun-Spot Measures," *Monthly Notices*, **65**, 813, 1905. Out of 4700 groups of spots observed from 1879 to 1901, during two eleven-year cycles, the Maunders have retained only 1872 groups, which were observed at least for six days. They obtained two different expressions for the rotation: the one derived from all the groups, each of which was treated individually with regard to its

transit across the disk; and the other from the spots of long duration observed at least on two successive transits:

From the first, $\xi = 875'.7 - 164' \sin^2 \lambda$.
From the second, $\xi = 866'.6 - 128' \sin^2 \lambda$.

The second of these formulae is in almost complete accord with that which we have derived from the movement of the faculae. It seems furthermore to deserve more confidence than the first formula, for, in the first place, it is less compromised than the first by the objections which may be raised against Mr. Maunder's method; in the second place, as that author remarked, "there is no doubt that these groups [long-lived groups] are much more free from the effect of accidental motions than the groups when considered separately at each apparition." But, in view of such different formulae to represent the same phenomenon, the observer is in much perplexity. Each formula represents the mean rotation of a certain number of spots, which may differ from the mean rotation of a certain number of other spots, and differs from the rotation of the photosphere. The last fact is the most important one to consider, for to make deductions from the rotation of a certain number of spots, it is necessary that the sum of their proper motions should be zero. We are consequently under the necessity of employing in this research only those spots which have small proper motions, under penalty of vitiating the result, and of comparing among themselves the heliographic positions of points that are well determined.

The conclusion of the comparison which we have attempted to make seems to us clearly to indicate that the rotation of the faculae does not differ more from the rotation of the spots than the rotations of different spots differ among themselves. To make a more accurate comparison of the rotations of these two superposed strata, it would be necessary to know both with a greater degree of certainty.

Comparison with the rotation determined from the flocculi.—There is only one preliminary work upon this rotation, that of Messrs. Hale and Fox, already cited. We are therefore able to compare with provisional results only. It is sufficient to say that it is necessary

that we should not be hasty in drawing conclusions. It will moreover be useful to place before the eyes of the reader the two results as we have arranged them in the following table:

Zones	Northern Hemisphere				Southern Hemisphere			
	Flocculi		Faculae		Flocculi		Faculae	
0° to 5° . .	14°72	±0°031	14°47	±0°071	14°57	±0.045	14°52	±0°076
5 to 10. . .	14.50	.027	14.45	.044	14.55	.030	14.42	.050
10 to 15. . .	14.34	.024	14.28	.039	14.39	.020	14.39	.036
15 to 20. . .	14.14	.025	14.21	.049	14.30	.028	14.25	.040
20 to 25. . .	14.13	.035	14.17	.088	14.11	.038	14.21	.051
25 to 30. . .	13.74	.060	13.90	.113	14.03	.073	14.04	.063
30 to 35. . .	13.64	.073	. . . . .	. . . . . .	13.93	.120	. . . . .	. . . . .

Zones	Means of Both Hemispheres			
	Flocculi		Faculae	
0° to 5°	14°66	±0°026	14°50	±0°033
5 to 10.	14.52	.020	14.44	.021
10 to 15.	14.37	.016	14.34	.017
15 to 20.	14.22	.019	14.24	.020
20 to 25.	14.12	.026	14.20	.032
25 to 30.	13.90	.049	14.00	.041
30 to 35.	13.76	.067	. . . . .	. . . .

According to these preliminary results the rotation of the flocculi would seem to be more rapid at the equator than that of the faculae; but their variation of velocity as a function of the latitude would be more much rapid than for the faculae.

Comparison with the rotation of the reversing layer.—The memoir by N. C. Dunér is still the most complete work upon the rotation of the reversing layer, at least to our knowledge. According to Dunér, the rotation of the absorbing stratum is well represented by the following formula:

$$\xi = 14^\circ\!.807 - 4^\circ\!.172 \sin^2 \lambda,$$

while we have found for the faculae

$$\xi = 14^\circ\!.470 - 2^\circ\!.268 \sin^2 \lambda.$$

The rotation of the reversing layer would therefore be still more rapid than that of the flocculi. As for the flocculi, the variation

as a function of the latitude is still more rapid than that which we have found.

The last two comparisons would favor the supposition that the acceleration derived from our measures may be a little too small, but its agreement with the formula which Mr. and Mrs. Maunder have derived from a study of spots of long duration is, on the contrary, favorable to its correctness. This question, therefore, still requires new investigations before it can be fully cleared up.

OBSERVATOIRE DE ZÔ-SÈ
CHINA

Reviews

Physik der Sonne. Von E. Pringsheim. Leipzig and Berlin: B. G. Teubner, 1910. 8vo, pp. 435, with 235 diagrams and 7 plates; cloth, M. 18.

Never in the history of astronomy has the interest in solar research been so great as at the present time. The perfection of the spectroscope, the invention of the spectroheliograph, the construction of great astrophysical observatories, and the activity of the International Union for Co-operation in Solar Research have all contributed largely toward this end. Great discoveries and new theories have followed each other so rapidly that it has been difficult to keep pace with them. Under these conditions any book on the subject must necessarily be somewhat out of date by the time it leaves the press. However, in the *Physik der Sonne* we find an excellent résumé of the status of the scientific knowledge of the sun down to the year 1909.

For many years Professor Pringsheim gave a course of lectures under the above title at the University of Berlin, and the book is the outcome of these lectures. They were open to students of all the faculties and were therefore popular in nature. The book likewise will appeal not only to astrophysicists and physicists, but to all who are interested in general science.

In the introduction the author deals with the sun as the source of all terrestrial energy and considers its influence upon the life on the earth. In successive chapters he discusses the distance, size, and mass of the sun; the photosphere; solar rotation and periodicity of solar activity; spectrum of the sun and its chemical composition; solar eclipses; chromosphere and prominences; solar theories; corona and solar atmosphere; flocculi, vortices, and Zeeman effect; and radiation and temperature of the sun.

For many readers the most interesting chapters of the book will be those that deal with the theories of the sun. "Die Theorien entstehen und vergehen wie die Blätter der Bäume und die Geschlechter der Menschen," as the author says. On account of their historic interest, several of the older theories are given in outline; e.g., the theories of Cossini, Wilson, Bode, Herschel, Kirchhoff, Zöllner, Respighi, and Lockyer; while the more modern theories of Young, Schmidt, Schwarz-

schild, Julius, Emden, and Oppolzer are discussed more in detail. The anomalous dispersion theory of Julius is given more attention than any other. The author says that this theory has always been taken more seriously by physicists than by astrophysicists. This is perhaps quite natural. It is very difficult for one who has observed sun-spots, faculae, flocculi, and prominences to accept a theory which explains them as mere illusions.

Interesting summaries of numerical data appear throughout the book. The latest value of the sun-spot period is given as 11.125 years, with sub-periods of 8.344 and 4.768 years. From a discussion of the values of the solar constant from the determination of Pouillet in 1837 down to that of Abbot and Fowle in 1908, a mean of 2.2 is derived, and from this an effective temperature of 6033° absolute.

Tables of the distribution of light, heat, spectral and chemical intensity over the sun's disk are given, as well as the transmission coefficients for different wave-lengths. From a combination of the determinations by different methods of the daily velocity of rotation, mean values are derived ranging from 14°7 at the equator to 11°8 at about latitude 75°. These values will be somewhat changed by the more recent investigations of Adams, Perot, and others.

The illustrations are for the most part poor, partly due to the poor paper upon which they are printed. Figs. 113 and 114 have been so retouched in the process of reproduction that they bear little resemblance to the originals.

Attention may be called to a few mistakes. The date of Fig. 101, p. 181, is May 28, 1900, not 1893 as stated on p. 182. To preserve the historic interest of Fig. 103, p. 182, the exact date July 9, 1891, should be added. Referring to the working of the Rumford spectroheliograph, the two slits are fixed and the exposure is made by moving the sun's image over the first slit and the photographic plate over the second, and not as stated at the foot of p. 187. Figs. 108, 109, 110, and 111 are all from photographs taken at the Kenwood Observatory, not at the Yerkes Observatory as stated on p. 189. In Fig. 116, p. 195, the use of C between D_3 and F, apparently to denote coronium, is misleading, if not actually incorrect. Pringsheim himself calls attention to the fact that the chromospheric line 1474 K has a wave-length of 5316.8 and is not identical with the chief coronal line, the wave-length of which is now given as 5303.3. In Riccò's original plate in the *Comptes Rendus* this line is marked *b*. Fig. 183, p. 311, is from a drawing by Professor William Harkness based on twelve photographs.

Throughout the work original sources are cited, forming an excellent bibliography of the whole subject.

FREDERICK SLOCUM

Les théories modernes du soleil. Par J. BOSLER. (*Encyclopédie scientifique.*) Paris: Octave Doin & Fils, 1910. Pp. 370, with 49 diagrams. Fr. 5.

It is a question whether the observations of any natural phenomena have ever been explained in a greater variety of ways than the observations made on the sun. According to the various theories the same phenomena may be explained by the principles of conventional circulation, chemical dissociation, thermodynamics, electrodynamics, refraction, radioactivity, anomalous dispersion, etc.

M. Bosler has set forth most of the different theories in a very elementary way, and, in some cases, has added comments on the merits of the theory. It is interesting to note how the development of the theories has paralleled the development of science, and the reader will undoubtedly feel that the development must proceed farther before a satisfactory theory is found.

The illustrations are, with only a few exceptions, very poor. Fig. 45, p. 321, should be inverted, the legends remaining as they are.

FREDERICK SLOCUM

The Spectroscope and Its Work. By H. F. NEWALL. London: Society for Promoting Christian Knowledge (New York Agent, E. S. Gorham), 1910. 12 mo, pp. 163, with 58 figures, eight half-tone plates, and a frontispiece of colored spectra. 2*s*. 6*d*.

This appears as one of the "Manuals of Elementary Science" of the society named above. It should fulfil its purpose admirably, for it gives a sketch of nearly the whole field in a clear and simple manner. Difficult as is the task of writing a treatise on a given specialized department of science, that of selecting the main points and then compressing them into a primer of 160 pages is in some ways a greater one.

The first thing that strikes the reader is the fresh method of presentation of what is to some a familiar subject: the author has used his own original ideas, without being over-influenced therein by the classical works of earlier days. The figures are new and appropriate: one misses the old-time woodcuts, some of which have done good service for nearly

threescore years since their first appearance in memoirs of the pioneers of spectroscopy.

The book is written also to encourage personal observation in a fascinating subject. The historical facts are not omitted, however, and room is found for quotations from Newton's memoir on optics of 1675. After briefly describing the essentials of the spectroscope in an early chapter, the author later reverts to the subject in a particularly useful chapter entitled "Increase of the Power of a Spectroscope." This will be of value to many besides beginners in spectroscopy.

Five chapters are devoted to the work of the spectroscope in its astronomical applications, and these necessarily have to be brief. Two valuable chapters are entitled "Law in the Spectrum," one dealing with the continuous spectrum, the other with bright-line spectra. An excellent brief exposition of diffraction and the measurement of wave-length is given in chap. xiv. The final chapter treats of variations in the spectra of gases and vapors.

The plates are good, and the colored frontispiece should help the general reader in understanding the fundamental differences of spectra. The book deserves a wide circulation and is further recommended as a good work for collateral reading by classes in general astronomy.

E. B. F.

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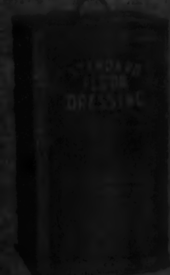

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